427L Handout: the multivariable chain rule

For the next three questions, we suppose that $x(t)$ and $y(t)$ are functions varying with t, with $x(t) = t^2 - t$ and $y(t) = e^t - 1$.

1. Suppose $f(x, y)$ is the function

$$f(x, y) = 3x + 5y + 6.$$

   Compute $\frac{df}{dt}$ at $t = 1$.

2. Suppose $g(x, y)$ is a function so that $\frac{\partial g}{\partial x}\big|_{(0,0)} = 0$ and $\frac{\partial g}{\partial y}\big|_{(0,0)} = 0$. Compute the derivative $\frac{dh}{dt}$ at $t = 1$, where

$$h(x, y) = 4x + y + 2 + g(x, y).$$

3. Suppose that $w(x, y)$ is a function whose graph passes through the point $P = (0, 0, 2)$ and whose tangent plane at P has equation

$$4x + y - z + 2 = 0.$$

   Compute the derivative $\frac{dw}{dt}$ at $t = 1$.

Now suppose that $x(s, t)$ and $y(s, t)$ are functions of s and t, with $x(s, t) = t^2 - st + \sin(\pi s)$ and $y(s, t) = e^t - te^s$.

4. Compute the partial derivatives of f, h, and w with respect to s and t, where f, h, and w are the functions from the first three questions above.