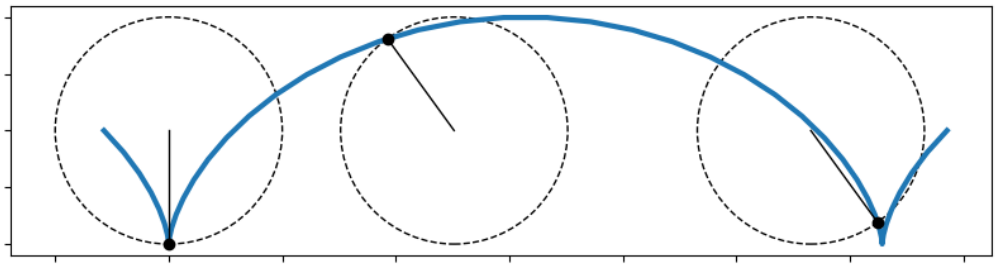
427L: Velocity, acceleration, and arc length

A *cycloid* is the curve traced out by a point on the edge of a circle as it rolls along a flat surface, like this:



If the circle has radius R, then the cycloid is parameterized by the equations

$$x(t) = Rt - R\sin(t), \quad y(t) = R - R\cos(t).$$

1. Find the velocity and acceleration vectors for this parameterization of the cycloid.

2. Find the maximum and minimum *speed* of a particle moving along this parameterization of the cycloid.

3. Find the length of one "arch" of the cycloid. (Hint: to solve this integral, try using the identity $1 - \cos(x) = 2\sin^2(x/2)$. Watch out for absolute values!)

4. Find a *unit-speed parameterization* for (some portion of) the cycloid.