

427L: Operations on vector fields

Let $f : \mathbb{R}^3 \rightarrow \mathbb{R}$ be a function and let $F, G : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be vector fields.

Which of the following operations actually make sense?

1. $\nabla \cdot f$
2. $\nabla \cdot F$
3. The curl of the divergence of f
4. The dot product of the divergence of G with F
5. $\nabla \times (\nabla f)$
6. The cross product of the curl of F with the gradient of f
7. $\nabla(\nabla f)$
8. $\nabla \times (F \cdot G)$
9. The curl of the curl of f
10. $\nabla \cdot (\nabla f)$
11. The divergence of the cross product of F and G
12. ∇F
13. The directional derivative of f along F

For the operations above which *do* make sense, write down a formula in terms of f , $F = (F_x, F_y, F_z)$, and $G = (G_x, G_y, G_z)$ (and their partial derivatives).