427L: integration over non-rectangular regions

For each region $R \subset \mathbb{R}^2$ given below, write down an iterated integral (or a *sum* of iterated integrals) giving the value of

$$\iint_R f \, dA,$$

where f is a function defined on R. That is, write down a *sum* of expressions in *one* of the two forms below:

$$\int_{x_1}^{x_2} \int_{y_1}^{y_2} f(x,y) \, dy \, dx, \qquad \int_{y_1}^{y_2} \int_{x_1}^{x_2} f(x,y) \, dx \, dy,$$

where in the first integral y_1, y_2 can depend on x, and in the second integral x_1, x_2 can depend on y.

1. R is the right triangle with vertices at $(0,0)$, $(4,0)$, and $(4,3)$.

2. R is the equilateral triangle with vertices at $(-1,0)$, $(1,0)$, and $(0,\sqrt{3})$.

3. R is the semicircular region given by the intersection of the circle with radius 2 centered at $(3,0)$ with the half-space $y \geq 0$.

4. R is the *annulus* (or "ring") centered at the origin with "inner radius" 1 and "outer radius" 4. (In polar coordinates, R is given by $\{(r,\theta) : 1 \leq r \leq 4\}$).

There are more problems on the back!

Change the order of integration on the following integrals:

1.

$$\int_0^9 \int_0^{\sqrt{y}} dx\, dy$$

2.

$$\int_{\pi/2}^{\pi} \int_0^{\sin x} dy\, dx$$