427L: integration over surfaces

1. Find a parameterization of the cylinder S of radius 2 whose axis lies along the vector $\langle 1, -2, 0\rangle$, and which lies between the planes $x - 2y = 0$ and $x - 2y = 15$. Then find the integral

$$\iint_S x + z \, dS.$$

2. Let $\mathbf{F}$ be the (constant) vector field $2\mathbf{k}$. Evaluate the integral

$$\iint_S \mathbf{F} \cdot d\mathbf{S},$$

where S is the surface from the previous question.

3. Let S be the part of the paraboloid with equation $z = x^2 + y^2 + 1$ lying between the planes $z = 1$ and $z = 10$, *and* lying in the half-space $x \geq 0$.

Sketch a picture of the paraboloid, and evaluate the integral

$$\iint_S x \, dS.$$

4. Let S be the closed surface bounded by the upper part of the hemisphere $x^2 + y^2 + z^2 = 1$, and by the disk $x^2 + y^2 = 1$, $z = 0$. Let $\mathbf{F}$ be the vector field $\mathbf{F} = 2x\mathbf{i} + 2y\mathbf{j} + 2z\mathbf{k}$. Find the integral of $\mathbf{F}$ over S (where S is oriented with outward-pointing normal).