Name: Answer Key

M427L Quiz (2/8/22)

A B C

1. Find a function $f(x,y)$ so the the plane P passing through the points $(-1,2,1)$, $(3,0,2)$, $(4,1,-1)$ is the graph of f. That is, find $f:\mathbb{R}^2 \to R$ so that $P = \{(x,y,z) \in \mathbb{R}^3 : z = f(x,y)\}$.

$$\vec{u} = \vec{AB} = \begin{pmatrix}3\\0\\2\end{pmatrix} - \begin{pmatrix}-1\\2\\1\end{pmatrix} = \begin{pmatrix}4\\-2\\1\end{pmatrix}, \quad \vec{v} = \vec{BC} = \begin{pmatrix}4\\1\\-1\end{pmatrix} - \begin{pmatrix}3\\0\\2\end{pmatrix} = \begin{pmatrix}1\\1\\-3\end{pmatrix}$$

Normal vector to P is $\vec{u}\times\vec{v} = \det\begin{pmatrix}\hat{i} & \hat{j} & \hat{k}\\ 4 & -2 & 1\\ 1 & 1 & -3\end{pmatrix} = \hat{i}\begin{vmatrix}-2 & 1\\ 1 & -3\end{vmatrix} - \hat{j}\begin{vmatrix}4 & 1\\ 1 & -3\end{vmatrix} + \hat{k}\begin{vmatrix}4 & -2\\ 1 & 1\end{vmatrix}$

$= 5\hat{i} + 13\hat{j} + 6\hat{k} = \vec{n}$. Eqn. for plane is $(p-A)\cdot\vec{n} = 0$, where $p = (x,y,z)$.

$\Rightarrow$ we get $5(x+1) + 13(y-2) + 6(z-1) = 0$

$5x + 13y + 6z + 5 - 26 - 6 = 0$

$5x + 13y + 6z - 27 = 0$

Solve for z:

$6z = -5x - 13y + 27$

$$\boxed{z = -\frac{5x}{6} - \frac{13y}{6} + \frac{27}{6}}$$

or $f(x,y) = -\frac{5x}{6} - \frac{13y}{6} + \frac{27}{6}$.

2. Sketch the region R described in *polar coordinates* by

$$R = \left\{(r,\theta) : \begin{array}{c} -\pi/2 < \theta < \pi/2 \\ 1 \le r \le \theta^2 + 2 \end{array}\right\}.$$

$\left(\frac{\pi}{2}\right)^2 \approx \frac{(3+.1)^2}{4} \approx \frac{9.6}{4} \approx 2.8$