Name: Solutions

427L Quiz (2/17/22)

1. Compute the limit, or show that it does not exist:

(a)

$$\lim_{x,y\to 0,0} \frac{x^4+xy^2}{x^2+y^2}$$

In polar coordinates:

$x = r\cos\theta$

$y = r\sin\theta$

$x^2+y^2 = r^2$

$$\lim_{(x,y)\to(0,0)} \frac{x^4+xy^2}{x^2+y^2} = \lim_{r\to 0} \frac{r^4\cos^4\theta + r\cos\theta \cdot r^2\sin^2\theta}{r^2}$$

$$= \lim_{r\to 0} r^2\cos^4\theta + r\cos\theta\sin^2\theta = \boxed{0}$$

(b)

$$\lim_{x,y\to 0,0} \frac{(x+y)^2-(x-y)^2}{xy}$$

Expand / Simplify: $(x+y)^2 - (x-y)^2 = x^2+2xy+y^2 - (x^2-2xy+y^2)$

$= 4xy$,

So limit is the same as $\lim_{(x,y)\to(0,0)} \frac{4xy}{xy} = \lim_{(x,y)\to 0,0} 4 = \boxed{4}$

2. Compute the partial derivatives f_x and f_y if $f(x,y) = 2x^2y - \sin(x)\cos(y) + e^{2y}$.

$$f_x = \frac{\partial}{\partial x}\left(2x^2y - \sin(x)\cos(y) + e^{2y}\right) = 4xy - \cos(x)\cos(y)$$

$$f_y = \frac{\partial}{\partial y}\left(2x^2y - \sin(x)\cos(y) + e^{2y}\right) = 2x^2 + \sin(x)\sin(y) + 2e^{2y}$$