Name: Solutions

427L Quiz (3/31/22)

1. Find the divergence and curl of the vector field

$$F(x,y,z) = \langle e^{xz}, \sin(xy), x^5y^3z^2\rangle.$$

divergence $= \nabla \cdot \vec{F} = \frac{\partial}{\partial x} e^{xz} + \frac{\partial}{\partial y}\sin(xy) + \frac{\partial}{\partial z} x^5y^3z^2$

$= ze^{xz} + x\cos(xy) + 2x^5y^3z$

Curl $= \nabla \times \vec{F} = \det\begin{pmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ e^{xz} & \sin(xy) & x^5y^3z^2 \end{pmatrix} = \vec{i}\left(\frac{\partial}{\partial y} x^5y^3z^2 - \frac{\partial}{\partial z}\sin(xy)\right)$

$- \vec{j}\left(\frac{\partial}{\partial x} x^5y^3z^2 - \frac{\partial}{\partial z} e^{xz}\right)$

$+ \vec{k}\left(\frac{\partial}{\partial x}\sin(xy) - \frac{\partial}{\partial y} e^{xz}\right)$

$= (3x^5y^2z^2)\vec{i} + (xe^{xz} - 5x^4y^3z^2)\vec{j} + (y\cos(xy))\vec{k}.$

2. Find the volume of the solid region bounded by: the surface $z = x^2 + y^4$, the xy-plane, the xz-plane, the yz-plane, and the planes $x = 1$ and $y = 2$.

Region lies between the surface $x^2+y^4 = z$ and the rectangle $[0,1]\times[0,2]$ in $\mathbb{R}^2$.

$$\int_0^2\int_0^1 x^2+y^4\,dx\,dy = \int_0^2 \frac{x^3}{3} + xy^4\Big|_0^1 dy$$

$$= \int_0^2 \frac{1}{3} + y^4\,dy = \frac{y}{3} + \frac{y^5}{5}\Big|_0^2 = \frac{2}{3} + \frac{32}{5}$$

$$= \frac{10}{15} + \frac{96}{15} = \frac{106}{15}$$