Name: Solutions

427L Quiz ~~(2/17~~ 4/14/22)

1. Find the volume of the solid lying above the disk $D = \{(x,y) : x^2 + y^2 \leq 9\}$ in $\mathbb{R}^2$, and below the surface in $\mathbb{R}^3$ with equation $z = e^{x^2+y^2}$.

In polar coordinates: $D = \{(r,\theta) : r \leq 3\}$, $z = e^{r^2}$

$$\iint_D e^{r^2}\, dA = \int_0^3 \int_0^{2\pi} e^{r^2}\, r\, d\theta\, dr = 2\pi \int_0^3 e^{r^2}\, r\, dr.$$

Substitute:
$u = r^2$
$du = 2r\,dr$
$\frac{du}{2} = r\,dr$

$$2\pi \int_{r=0}^{r=3} e^u \cdot \frac{du}{2} = \pi \cdot e^u \Big|_{r=0}^{r=3} = \pi\left(e^9 - 1\right).$$

2. Use the change of coordinates $x = u$ and $y = 2u + v$ to rewrite the integral

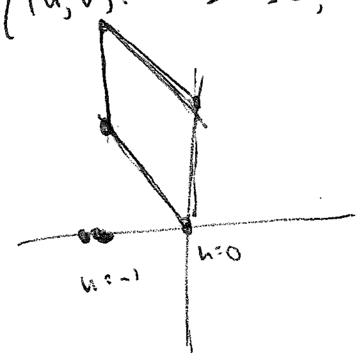
$$\int_0^2 \int_{-1}^0 x^2 + xy^3\, dx\, dy$$

as an integral in (u,v) coordinates. You do *not* need to evaluate the integral. (Hint: the region of integration in u, v coordinates should *not* be a rectangle. What is the right region of integration? What order of integration is easiest?)

When $y = 0$,
$v = -2u$.
when $y = 2$
$v = 2 - 2u$.

In (u,v) coordinates: $[-1,0] \times [0,2]$ is region

$\{(u,v) : -1 \leq u \leq 0,\ -2u \leq v \leq 2 - 2u\}$.



Jacobian is $\begin{pmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$

Jacobian determinant is $\det\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} = 1.$

$$\int_{-1}^{0} \int_{-2u}^{2-2u} u^2 + u\left(2u+v\right)^3 \cdot 1\, dv\, du$$