M427L: Exam 2 review

Chapter 4

1. The acceleration, initial velocity, and initial position of a particle traveling through space are given by
$$\vec{a}(t) = \langle 2, -6, -4\rangle, \quad \vec{v}(0) = \langle -5, 1, 3\rangle, \quad \vec{r}(0) = (6, -2, 1).$$
The particle's path intersects the the yz plane at exactly two points. Find those two points.

2. If $c(t)$ is the *helix* $c(t) = (\cos t, \sin t, 4t)$, find a function $\ell(s)$ representing the length of the curve c from $t = 0$ to $t = s$.

3. Sketch a vector field whose curl is not the zero function *and* whose divergence is not the zero function. Write down an equation for a vector field (possibly not the same one) which satisfies the same properties.

4. Write down a formula for $\nabla\cdot(f\vec{F})$, where $f : \mathbb{R}^3 \to \mathbb{R}$ is a function and $\vec{F} : \mathbb{R}^3 \to \mathbb{R}^3$ is a vector field. (You can write this down in terms of f and its partial derivatives, and $\vec{F} = (F_1, F_2, F_3)$ and the partial derivatives of these quantities).

Chapter 5

1. Evaluate the integral
$$\iint_R (xy)^2 \cos x^3 \; dA,$$
where R is the rectangle $[0, \pi] \times [0, 1]$.

2. Let D be the region of $\mathbb{R}^2$ given by the half-disk centered at $(0, 2)$ with radius 1, to the right of the y-axis. Evaluate the integral
$$\iint_D (y - 2) \cdot x \; dA.$$

3. Let R be the region in $\mathbb{R}^3$ bounded by the coordinate planes (the xy, yz, and xz planes) and the plane $2x + 2y + z = 5$. Evaluate the integral
$$\iiint_R x^2 z - 2yz^2 \; dV.$$

4. Evaluate the integral
$$\int_0^4 \int_{y/2}^2 e^{x^2} \; dx \, dy$$
by changing the order of integration.

Chapter 6

1. Let $T(u, v) = (u^2 - v^2, 2uv)$, and let D' be the region of $\mathbb{R}^2$ given by $\{(u, v) : u^2 + v^2 \leq 1, u \geq 0, v \geq 0\}$. Describe the region $D = T(D')$, and evaluate

$$\iint_D dx\, dy.$$

2. Let A be the annulus $\{(x, y) : 1 \leq (x^2 + y^2) \leq 4\}$. Find the integral

$$\iint_A xy + y^2 \; dx\, dy.$$

3. Find the volume of the solid in $\mathbb{R}^3$ bounded below by the paraboloid $z = x^2 + y^2$ and above by the cone $z = \sqrt{x^2 + y^2}$.

4. Let E be the ellipsoid

$$\frac{x^2}{2} + \frac{y^2}{3} + z^2 = 1.$$

Evaluate the integral

$$\iiint_E \frac{xy + z}{3} \; dV.$$

Chapter 7

1. Evaluate the path integral

$$\int_c f(x, y, z) \; ds$$

where $c : [0, \pi] \to \mathbb{R}^3$ is the curve $t \mapsto (\sin t, \cos t, t)$ and $f(x, y, z) = x + y + z$.

2. Let C be the boundary of the unit square $[0, 1] \times [0, 1]$, oriented counterclockwise, and let F be the vector field $y^2\mathbf{i} - xy\mathbf{j}$. Evaluate the line integral

$$\int_C F \cdot d\mathbf{r}.$$