M427L: final review

Note: this review sheet only has problems from the *last* part of the semester (so it does *not* have any material covered on the previous review sheets). You are still responsible for *all* of the material from this class on the final, so you may want to go back and look at earlier review sheets!

Chapter 7

1. Determine if the vector field
$$\vec{F}(x,y,z) = 2ye^{2xy}\vec{i} + (2xe^{2xy} + 6yz)\vec{j} + 3y^2\vec{k}$$
is the gradient of some function $f : \mathbb{R}^3 \to \mathbb{R}$. If it is, find an f so that $\vec{F} = \nabla f$.

2. Let C be the curve given by the graph of the function $y = 2x^2$, where x ranges from 0 to 1, and let $\vec{F}$ be the vector field
$$\vec{F}(x,y) = (x^2 - y^2)\vec{i} + (x^2 + y^2)\vec{j}.$$
Evaluate the line integral
$$\int_C \vec{F} \cdot dr.$$

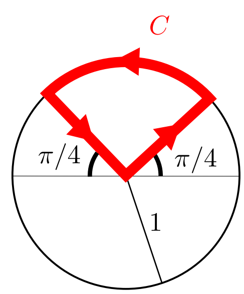
3. Find the area of the surface S given by the intersection of the unit sphere with the cone $z \geq \sqrt{3(x^2 + y^2)}$.

4. Let S be the closed surface given by a cone of height 4, whose axis lies along the z axis and whose base lies in the xy plane and has radius 3. Write down (but do not evaluate) iterated integrals you could use to find the surface integral
$$\iint_S (3x + 2y)\, dA.$$

5. Let S be the portion of graph of the function $z = 3x^2 - 2xy$ above the rectangle $[0,1] \times [0,2]$, oriented with upward-pointing normal. Find the flux of the vector field
$$\vec{F}(x,y,z) = (x - y)\vec{i} + xz\vec{j} - 2yz\vec{k}$$
through the surface S.

Chapter 8

1. Let C be the curve bounding a wedge-shaped region, pictured in red below:



Use Green's theorem to find the integral

$$\int_C \vec{F} \cdot dr,$$

where $\vec{F}$ is the vector field $\vec{F}(x, y) = x^2 y\vec{i} - (2x + y)\vec{j}$.

2. Let C be the closed, piecewise smooth curve formed by traveling in straight lines between the points $(0, 0, 0)$, $(2, 1, 5), (1, 1, 3)$ in that order. Use Stokes' theorem to evaluate the integral

$$\int_C (xyz)\ dx + (xy)\ dy + (x)\ dz.$$

3. Use the Divergence Theorem to find the flux of the vector field

$$\vec{F}(x, y, z) = (x - y^2)\vec{i} + y\vec{j} + x^3\vec{k}$$

out of the rectangular solid $[0, 1] \times [1, 2] \times [1, 4]$.

4. (bonus) Let $\vec{F}(x, y, z)$ be the vector field $\vec{i} + \vec{j} - \vec{k}$. Find the flux of $\vec{F}$ through the surface S given by the part of the graph of $e^{2x-y}\sqrt{1 - x^2 - y^2}$ over the unit disk $x^2 + y^2 \leq 1$, oriented with upward-pointing normal. (Hint: is $\vec{F}$ the curl of a vector field? Why would this help?)