427L: Rules for vector operations

In this class we will use four basic operations on vectors in $\mathbb{R}^n$. Some of them also involve *scalars* (another name for real numbers).

1. *Vector addition*: we can take two *vectors* $\vec{u}$, $\vec{v}$ and add them together to get a third *vector* $\vec{u}+\vec{v}$.
2. *Scalar multiplication*: we can take one *vector* $\vec{u}$ and one *scalar* $a \in \mathbb{R}$ and multiply them to get another *vector* $a\vec{u}$.
3. *Dot product*, also called *inner product* or sometimes *scalar product*: we can take two *vectors* $\vec{u}$, $\vec{v}$ and combine them to get a *scalar* $\vec{u}\cdot\vec{v}$.
4. *Cross product*: we can take two *vectors* $\vec{u}$, $\vec{v}$ in $\mathbb{R}^3$ and combine them to get a *vector* $\vec{u}\times\vec{v}$.

These operations all obey certain rules. **You should use the definitions of the four operations given above to check that these rules are satisfied!** It will give you a better feel both for the definitions and for how to manipulate vectors. In particular, **do not try to learn this sheet by memorizing the rules.** It will be *much* more effective to just practice working with vectors until you get a feel for how they behave. (For this reason I am not listing all of the different properties of dot products and cross products on this sheet either).

Basic rules (scalar multiplication and vector addition)

1. *Vector addition commutes:* if $\vec{u}$ and $\vec{v}$ are vectors, then $\vec{u}+\vec{v}=\vec{v}+\vec{u}$.
2. *Vector addition is associative*: if $\vec{u}$, $\vec{v}$, $\vec{w}$ are vectors, then $(\vec{u}+\vec{v})+\vec{w}=\vec{u}+(\vec{v}+\vec{w})$.
3. *Scalar multiplication distributes, I:* if $a \in \mathbb{R}$ and $\vec{u},\vec{v}$ are vectors, then $a(\vec{u}+\vec{v})=a\vec{u}+a\vec{v}$.
4. *Scalar multiplication distributes, II:* if $a,b \in \mathbb{R}$ and $\vec{u}$ is a vector, then $(a+b)\vec{u}=a\vec{u}+b\vec{u}$.
5. *Scalar multiplication is associative:* if a,b are scalars, and $\vec{v}$ is a vector, then $a(b\vec{v})=(ab)\vec{v}$.

Rules for the dot product

1. *Dot product is commutative*: if $\vec{u}$ and $\vec{v}$ are vectors, then $\vec{u}\cdot\vec{v}=\vec{v}\cdot\vec{u}$.
2. *Dot products distribute over sums*: if $\vec{u}$, $\vec{v}$, $\vec{w}$ are all vectors, then $\vec{u}\cdot(\vec{v}+\vec{w})=\vec{u}\cdot\vec{v}+\vec{u}\cdot\vec{w}$.
3. *Dot products and scalar multiplication*: if a is a scalar and $\vec{u}$, $\vec{v}$ are scalars, then $(a\vec{u})\cdot\vec{v}=a(\vec{u}\cdot\vec{v})$.

Rules for the cross product

1. *Cross product is* ***anti****commutative*: if $\vec{u}$ and $\vec{v}$ are vectors, then $\vec{u}\times\vec{v}=-(\vec{v}\times\vec{u})$.
2. *Cross products and sums*: if $\vec{u}$, $\vec{v}$, and $\vec{w}$ are vectors, then $\vec{u}\times(\vec{v}+\vec{w})=\vec{u}\times\vec{v}+\vec{u}\times\vec{w}$.
3. *Cross products and scalars*: if a is a scalar and $\vec{u}$, $\vec{v}$ are vectors, then $(a\vec{u})\times\vec{v}=a(\vec{u}\times\vec{v})$.

Warning: the cross product is *not* associative. This means that it is *not* true that $(\vec{u}\times\vec{v})\times\vec{w}=\vec{u}\times(\vec{v}\times\vec{w})$.