Hyperbolic groups summer mini-course: day 4 exercises
Algorithmic properties of hyperbolic groups

August 15, 2019

1. Show that in a δ-hyperbolic metric space, geodesic n-gons are $\delta(n-2)$-thin. That is, every side of the n-gon is within a $\delta(n-2)$ neighborhood of the union of the remaining sides.

2. Show that $\mathbb{Z}^2$ has quadratic Dehn function. Find an algorithm to solve the word problem for $\mathbb{Z}^2$ in (asymptotically) less than quadratic time.

3. Find a (possibly very vague) description of an algorithm to find a Dehn presentation for a δ-hyperbolic group, given an arbitrary finite presentation. (You may assume that δ is known.)

   Conclude that there is a semi-decidable[1] algorithm that determines if a given finite group presentation presents a hyperbolic group. (Here you cannot assume you know δ ahead of time!)

4. For a finitely generated group Γ, and an element $\gamma \in \Gamma$, let the *negative cone type* of γ be the set

$$C^-(\gamma) = \{\eta \in \Gamma : |\eta\gamma| = |\gamma| + |\eta|\}.$$

   Show that there is a bijection between the cone types of Γ and the negative cone types of Γ. In particular, the set of negative cone types of a hyperbolic group Γ is finite.

5. Draw the cone type graph for F_2.

6. Find all of the cone types for $\mathbb{Z}^2$ with its standard presentation $\langle a, b \mid aba^{-1}b^{-1}\rangle$, and draw the geodesic automaton.

[1] a *semi-decidable* algorithm is an algorithm to solve a decision problem that halts if the answer is yes, but runs forever if the answer is no.