

Heegaard diagrams and symmetric products

1. Find a genus 4 Heegaard diagram for $S^2 \times S^1$.
2. Find a Heegaard diagram for \mathbb{RP}^3 .
3. Let Y be a closed oriented 3-manifold with a Heegaard diagram given by a genus g surface Σ and two sets of attaching curves: $\{\alpha_1, \dots, \alpha_g\}$ and $\{\beta_1, \dots, \beta_g\}$. Show that $H_1(Y) \cong H_1(\Sigma) / \langle [\alpha_1], \dots, [\alpha_g], [\beta_1], \dots, [\beta_g] \rangle$.
4. The Heegaard genus of a closed, oriented 3-manifold Y is the minimum genus of a surface Σ such that Y has a Heegaard decomposition (Σ, α, β) . For each positive integer g , find a 3-manifold whose Heegaard genus is g .