Conventions for Gamma Matrices

Our conventions for the four dimensional discussion are such that we use real four component spinors (when their indices are in an up position). Our choice of Minkowski metric is the 'mostly plus metric.'

We use the outer product to write our 4 x 4 matrices in terms of 2 x 2 matrices. If M and N are two such matrices where

$$M = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \quad , \quad N = \begin{pmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{pmatrix} \tag{1}$$

then we choose our conventions so that

$$\begin{aligned} M \otimes N &= \begin{pmatrix} m_{11} \begin{pmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{pmatrix} & m_{12} \begin{pmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{pmatrix} \\ m_{21} \begin{pmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{pmatrix} & m_{22} \begin{pmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{pmatrix} \end{pmatrix} \\ &= \begin{pmatrix} m_{11}n_{11} & m_{11}n_{12} & m_{12}n_{11} & m_{12}n_{12} \\ m_{11}n_{21} & m_{11}n_{22} & m_{12}n_{21} & m_{12}n_{22} \\ m_{21}n_{11} & m_{21}n_{12} & m_{22}n_{11} & m_{22}n_{12} \\ m_{21}n_{21} & m_{21}n_{22} & m_{22}n_{21} & m_{22}n_{22} \end{pmatrix} \quad . \end{aligned} \tag{2}$$

In these conventions, there are sixteen real matrices we define by

$$\begin{aligned} &(\mathbf{I}_4)_a{}^b = (\mathbf{I}_2 \otimes \mathbf{I}_2)_a{}^b \quad , \\ &(\gamma^0)_a{}^b = i(\sigma^3 \otimes \sigma^2)_a{}^b \quad , \quad (\gamma^1)_a{}^b = (\mathbf{I}_2 \otimes \sigma^1)_a{}^b \quad , \\ &(\gamma^2)_a{}^b = (\sigma^2 \otimes \sigma^2)_a{}^b \quad , \quad (\gamma^3)_a{}^b = (\mathbf{I}_2 \otimes \sigma^3)_a{}^b \quad , \\ &(\gamma^0\gamma^1)_a{}^b = (\sigma^3 \otimes \sigma^3)_a{}^b \quad , \quad (\gamma^0\gamma^2)_a{}^b = (\sigma^1 \otimes \sigma^1)_a{}^b \quad , \quad (\gamma^0\gamma^3)_a{}^b = -(\sigma^3 \otimes \sigma^1)_a{}^b \quad , \\ &(\gamma^1\gamma^2)_a{}^b = i(\sigma^2 \otimes \sigma^3)_a{}^b \quad , \quad (\gamma^2\gamma^3)_a{}^b = i(\sigma^2 \otimes \sigma^1)_a{}^b \quad , \quad (\gamma^3\gamma^1)_a{}^b = i(\mathbf{I}_2 \otimes \sigma^2)_a{}^b \quad , \\ &(\gamma^0\gamma^1\gamma^2)_a{}^b = -(\sigma^1 \otimes \sigma^1)_a{}^b \quad , \quad (\gamma^1\gamma^2\gamma^3)_a{}^b = i(\sigma^2 \otimes \mathbf{I}_2)_a{}^b \quad , \\ &(\gamma^2\gamma^3\gamma^0)_a{}^b = i(\sigma^2 \otimes \sigma^3)_a{}^b \quad , \quad (\gamma^3\gamma^1\gamma^0)_a{}^b = -(\sigma^3 \otimes \mathbf{I}_2)_a{}^b \quad , \\ &(\gamma^5)_a{}^b = i(\gamma^0\gamma^1\gamma^2\gamma^3)_a{}^b = -(\sigma^1 \otimes \sigma^2)_a{}^b \quad . \end{aligned} \tag{3}$$

In order to raise and lower spinor indices, we define a spinor metric by

$$C_{ab} \equiv -i(\sigma^3 \otimes \sigma^2)_{ab} \;=\; \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix} \quad \to \quad C_{ab} = -C_{ba} \quad . \tag{4}$$

The inverse spinor metric is defined by the condition $C^{ab}C_{ac} = \delta_c{}^b$.

The spinor metric can be used to lower the second index on all sixteen of the matrices leading to the results

$$\begin{aligned}
&C_{ab} = -i(\sigma^3 \otimes \sigma^2)_{ab} \quad , \\[2ex]
&(\gamma^0)_{ab} = i(\mathbf{I}_2 \otimes \mathbf{I}_2)_{ab} \quad , \quad (\gamma^1)_{ab} = (\sigma^3 \otimes \sigma^3)_{ab} \quad , \\
&(\gamma^2)_{ab} = (\sigma^1 \otimes \sigma^1)_{ab} \quad , \quad (\gamma^3)_{ab} = -(\sigma^3 \otimes \sigma^1)_{ab} \quad , \\[2ex]
&(\gamma^0\gamma^1)_{ab} = -(\mathbf{I}_2 \otimes \sigma^1)_{ab} \quad , \quad (\gamma^0\gamma^2)_{ab} = -(\sigma^2 \otimes \sigma^2)_{ab} \quad , \quad (\gamma^0\gamma^3)_{ab} = -(\mathbf{I}_2 \otimes \sigma^3)_{ab} \quad , \\
&(\gamma^1\gamma^2)_{ab} = (\sigma^1 \otimes \sigma^1)_{ab} \quad , \quad (\gamma^2\gamma^3)_{ab} = -i(\sigma^2 \otimes \sigma^3)_{ab} \quad , \quad (\gamma^3\gamma^1)_{ab} = (\sigma^3 \otimes \mathbf{I}_2)_{ab} \quad , \\[2ex]
&(\gamma^0\gamma^1\gamma^2)_{ab} = i(\sigma^2 \otimes \sigma^3)_{ab} \quad , \quad (\gamma^1\gamma^2\gamma^3)_{ab} = i(\sigma^1 \otimes \sigma^2)_{ab} \quad , \\
&(\gamma^2\gamma^3\gamma^0)_{ab} = i(\sigma^2 \otimes \sigma^1)_{ab} \quad , \quad (\gamma^3\gamma^1\gamma^0)_{ab} = i(\mathbf{I}_2 \otimes \sigma^2)_{ab} \quad , \\[2ex]
&(\gamma^5)_{ab} = -i(\gamma^0\gamma^1\gamma^2\gamma^3)_{ab} = -(\sigma^2 \otimes \mathbf{I}_2)_{ab} \quad .
\end{aligned} \tag{5}$$

There equations imply the following properties

$$\begin{aligned}
&(\gamma^\mu)_{ab} = +(\gamma^\mu)_{ba} \quad , \quad (\gamma^\mu\gamma^\nu)_{ab} = +(\gamma^\mu\gamma^\nu)_{ba} \quad , \\
&(\gamma^\mu\gamma^\nu\gamma^\rho)_{ab} = -(\gamma^\mu\gamma^\nu\gamma^\rho)_{ba} \quad , \quad (\gamma^5)_{ab} = -(\gamma^5)_{ba} \quad .
\end{aligned}$$

where μ, ν, and ρ take on the values of 0, 1, 2, and 3.

Some useful Identities then follow

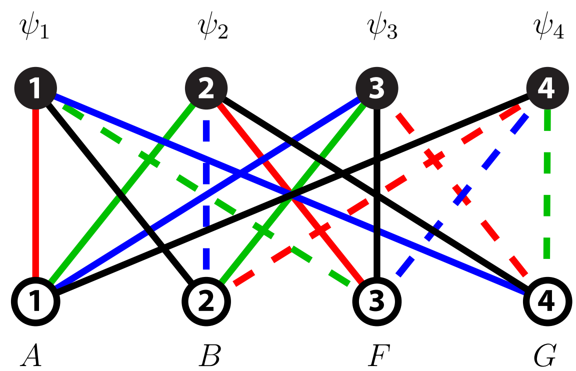
$$\begin{aligned}
&\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2\eta^{\mu\nu} \mathbf{I}_4 \quad , \quad \gamma^\mu \gamma_\mu = 4\mathbf{I}_4 \quad , \quad \gamma^\mu \gamma_\alpha \gamma_\mu = -2\gamma_\alpha \quad , \\
&\gamma^5 [\gamma^\alpha , \gamma^\beta] = -i\tfrac{1}{2} \epsilon^{\alpha\beta\mu\nu} [\gamma_\mu , \gamma_\nu] \quad , \quad \gamma^\mu [\gamma^\alpha , \gamma^\beta] \gamma_\mu = 0 \quad , \\
&\gamma^\mu [\gamma^\alpha , \gamma^\beta] = 2[\eta^{\mu\alpha}\gamma^\beta - \eta^{\mu\beta}\gamma^\alpha] + i2\epsilon^{\alpha\beta\mu\nu}\gamma^5\gamma_\nu \quad , \\
&[\gamma^\alpha , \gamma^\beta]\gamma^\mu = -2[\eta^{\mu\alpha}\gamma^\beta - \eta^{\mu\beta}\gamma^\alpha] + i2\epsilon^{\alpha\beta\mu\nu}\gamma^5\gamma_\nu \quad .
\end{aligned} \tag{6}$$

Chiral Supermultiplet 0-Brane Valise Formulation

We use the symbol D_a interchangeably with Q_a in the following equations for the valise formulation of the chiral supermultiplet

$$
\begin{aligned}
\mathrm{D}_a A &= \psi_a \quad , \quad \mathrm{D}_a B = i\,(\gamma^5)_a{}^b \psi_b \quad , \\
\mathrm{D}_a F &= (\gamma^0)_a{}^b\,\psi_b \quad , \quad \mathrm{D}_a G = i\,(\gamma^5\gamma^0)_a{}^b\,\psi_b \quad , \\
\mathrm{D}_a \psi_b &= i\,(\gamma^0)_{ab}\,(\,\partial_\tau A\,) - (\gamma^5\gamma^0)_{ab}\,(\,\partial_\tau B\,) \\
&\quad - iC_{ab}\,(\,\partial_\tau F\,) + (\gamma^5)_{ab}\,(\,\partial_\tau G\,) \quad ,
\end{aligned}
$$

In these equations, the bosonic 'fields' A, B, F, and G only depend on 'time' τ. As well the fermionic fields ψ_1, ψ_2, ψ_3, and ψ_4 also only depend on time. The adinkra representation of these equations is given by

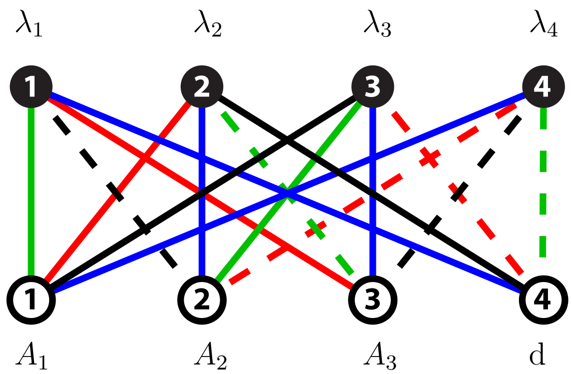


Vector Supermultiplet 0-Brane Valise Formulation

We use the symbol D_a interchangeably with Q_a in the following equations for the valise formulation of the vector supermultiplet

$$
\begin{aligned}
\mathrm{D}_a A_i &= (\gamma_i)_a{}^b \lambda_b \quad , \quad \mathrm{D}_a \mathrm{d} = i(\gamma^5\gamma^0)_a{}^b\,\lambda_b \quad , \\
\mathrm{D}_a \lambda_b &= -\,i\,(\gamma^0\gamma^i)_{ab}\,(\,\partial_\tau A_i\,) + (\gamma^5)_{ab}\,(\,\partial_\tau \mathrm{d}\,) \quad .
\end{aligned}
$$

In these equations, the bosonic 'fields' A_1, A_2, A_3, and d only depend on 'time' τ. As well the fermionic fields λ_1, λ_2, λ_3, and λ_4 also only depend on time. The adinkra representation of these equations is given by

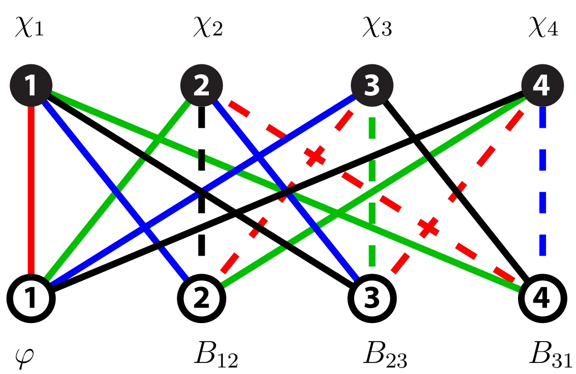


Tensor Supermultiplet 0-Brane Valise Formulation

We use the symbol D_a interchangeably with Q_a in the following equations for the valise formulation of the vector supermultiplet

$$\mathrm{D}_a\varphi = \chi_a \quad , \quad \mathrm{D}_a B_{i\,j} = -\tfrac{1}{4}([\gamma_i,\, \gamma_j])_a{}^b \chi_b \quad ,$$
$$\mathrm{D}_a\chi_b = i(\gamma^0)_{ab}\,\partial_\tau\varphi - (\gamma^5\gamma^i)_{ab}\,\epsilon_i{}^{j\,k}\partial_\tau B_{j\,k} \quad .$$

In these equations, the bosonic ‘fields’ φ, B_{12}, B_{23}, and B_{31} only depend on ‘time’ τ. As well the fermionic fields χ_1, χ_2, χ_3, and χ_4 also only depend on time.



On each field in each of the supermultiplets, the following condition is satisfied.

$$\mathrm{D}_a\,\mathrm{D}_b \;+\; \mathrm{D}_b\,\mathrm{D}_a \;=\; i2\,(\gamma^0)_{ab}\,\partial_\tau$$

which is the necessary condition to show these are representations of supersymmetry or SUSY.

General 1D Valise Formulation

Each of the multiplets discussed above can be re-written in the form of equations

$$\mathrm{D_I}\Phi_i \;= i\,(\mathrm{L_I})_{i\hat{k}}\Psi_{\hat{k}} \quad \text{and} \quad \mathrm{D_I}\Psi_{\hat{k}} \;= (\mathrm{R_I})_{\hat{k}i}\,(\partial_\tau \Phi_i) \quad ,$$

where Φ_i denote all the bosons, $\Psi_{\hat{k}}$ denotes all of the fermions. As well $(\mathrm{L_I})_{i\hat{k}}$ and $(\mathrm{R_I})_{\hat{k}i}$ are constant matrices. These real matrices are **not** γ-matrices and are found to satisfy the following relations

$$(\,\mathrm{L_I}\,)_i{}^{\hat{\jmath}}\,(\,\mathrm{R_J}\,)_{\hat{\jmath}}{}^k + (\,\mathrm{L_J}\,)_i{}^{\hat{\jmath}}\,(\,\mathrm{R_I}\,)_{\hat{\jmath}}{}^k = 2\,\delta_{\mathrm{IJ}}\,\delta_i{}^k \quad ,$$
$$(\,\mathrm{R_J}\,)_{\hat{\imath}}{}^j\,(\,\mathrm{L_I}\,)_j{}^{\hat{k}} + (\,\mathrm{R_I}\,)_{\hat{\imath}}{}^j\,(\,\mathrm{L_J}\,)_j{}^{\hat{k}} = 2\,\delta_{\mathrm{IJ}}\,\delta_{\hat{\imath}}{}^{\hat{k}} \quad .$$

$$(\,\mathrm{R_I}\,)_{\hat{\jmath}}{}^k\,\delta_{ik} = (\,\mathrm{L_I}\,)_i{}^{\hat{k}}\,\delta_{\hat{\jmath}\hat{k}} \quad ,$$

which we named as defining the "Garden Algebras." The L-matrices and R-matrices have a standard graph-theoretical interpretation. If we take the absolute value of the entries in all of these, then the resulting matrices describe the adjacency matrices of each adinkra diagram.

The Garden Algebra conditions ensure that

$$\{\,\mathrm{D_I}\,,\;\mathrm{D_J}\,\}\Phi_i \;=\; i2\,\partial_\tau\Phi_i \quad , \quad \{\,\mathrm{D_I}\,,\;\mathrm{D_J}\,\}\Psi_{\hat{k}} \;=\; i2\,\partial_\tau\Psi_{\hat{k}} \quad .$$

Looking At Simple CM-CM Quadratic Invariant

Let us examine the case of a quadratic constructed from the fields of a chiral supermultiplet in its valise formulation. Define $\mathcal{L}_1$ by

$$\mathcal{L}_1 \;=\; [\; A\,(\partial_\tau F) \;+\; \kappa_0 B\,(\partial_\tau G) \;+\; i\,\kappa_1 \tfrac{1}{2}\,\psi_a \mathcal{M}^{a\,b}\psi_b\,]$$

and let us calculate its derivative with respect to D_c. In this expression $\mathcal{M}^{a\,b}$ is a 4 $\times$ 4 matrix while κ_0 and κ_1 are constants. We find (and ignoring total derivatives in the calculation)

$$
\begin{aligned}
\mathrm{D}_c\mathcal{L}_1 \;&=\; \mathrm{D}_c[\; A\,(\partial_\tau\, F) \;+\; \kappa_0 B\,(\partial_\tau\, G) \;+\; i\,\kappa_1 \tfrac{1}{2}\,\psi_a \mathcal{M}^{a\,b}\psi_b\;] \\
\\
&=\; [\;(\mathrm{D}_c A)\,(\partial_\tau\, F) \;-\; (\partial_\tau\, A)\,(\,\mathrm{D}_c F) \\
&\qquad +\; \kappa_0(\mathrm{D}_c\, B)\,(\partial_\tau\, G) \;-\; \kappa_0(\partial_\tau B)\,(\mathrm{D}_c\, G) \\
&\qquad +\; i\,\kappa_1\,(\mathrm{D}_c\,\psi_a)\,\mathcal{M}^{a\,b}\psi_b\;] \\
&=\; [\;\psi_c\,(\partial_\tau\, F) \;-\; (\partial_\tau\, A)\,(\gamma^0)_c{}^d\psi_d \\
&\qquad +\; i\,\kappa_0\,(\gamma^5)_c{}^d\psi_d\,(\partial_\tau\, G) \;-\; i\kappa_0(\partial_\tau B)\;(\gamma^5\gamma^0)_c{}^d\psi_d\;] \\
&\qquad +\; i\,\kappa_1\,[\,i\,(\gamma^0)_{c\,a}\;(\partial_\tau A) \;-\; (\gamma^5\gamma^0)_{c\,a}\;(\partial_\tau B)\;]\mathcal{M}^{a\,d}\psi_d \\
&\qquad +\; i\,\kappa_1\,[\,-\,i\,C_{c\,a}\,(\partial_\tau\, F) \;+\; (\gamma^5)_{c\,a}(\partial_\tau\, G)\,]\mathcal{M}^{a\,d}\psi_d \\
&=\; [\;\psi_c\,(\partial_\tau\, F) \;-\; (\partial_\tau\, A)\,(\gamma^0)_c{}^d\psi_d \\
&\qquad +\; i\,\kappa_0\,(\gamma^5)_c{}^d\psi_d\,(\partial_\tau\, G) \;-\; i\kappa_0(\partial_\tau B)\;(\gamma^5\gamma^0)_c{}^d\psi_d\;] \\
&\qquad +\; [\,\kappa_1\,C_{c\,a}\,(\partial_\tau\, F) \;+\; i\,\kappa_1\,(\gamma^5)_{c\,a}(\partial_\tau\, G)\,]\mathcal{M}^{a\,d}\psi_d \\
&\qquad +\; [\,-\,\kappa_1\,(\gamma^0)_{c\,a}\;(\partial_\tau A) \;-\; i\kappa_1\,(\gamma^5\gamma^0)_{c\,a}\;(\partial_\tau B)\;]\mathcal{M}^{a\,d}\psi_d
\end{aligned}
$$

Let $\mathcal{M}^{ad} = C^{ad}$ and this becomes,

$$
\begin{aligned}
\mathrm{D}_c\mathcal{L}_1 \;=\; [\;(\,1\,-\,\kappa_1\,)\,\psi_c\,(\partial_\tau\, F) \;-\; (\,1\,-\,\kappa_1\,)\,(\partial_\tau\, A)\,(\gamma^0)_c{}^d\psi_d \\
+\; i(\,\kappa_0\,-\,\kappa_1\,)(\gamma^5)_c{}^d\psi_d\,(\partial_\tau\, G) \;-\; i(\,\kappa_0\,-\,\kappa_1\,)(\partial_\tau B)\;(\gamma^5\gamma^0)_c{}^d\psi_d\;]
\end{aligned}
$$

so that up to total derivatives we have

$$
\mathrm{D}_c\mathcal{L}_1 \;=\; 0 \quad .
$$

if $\kappa_0 \;=\; \kappa_1 \;=\; 1$. In the above calculations, we have freely used the identity

$$
\mathcal{X}\;(\partial_\tau\mathcal{Y}) \;\;=\;\; -\,(\partial_\tau\mathcal{X})\;\mathcal{Y} \;+\; \partial_\tau\,(\mathcal{X}\,\mathcal{Y})
$$

for any functions that depend only on τ.

Four Dimensional Supermultiplets

For four dimensional theories, all fields are allowed to depend on t, x, y, and z. In four dimensions, the form of the 'super D-equations' are slightly different for the chiral supermultiplet

$$
\begin{aligned}
{\rm D}_a A ~&=~ \psi_a ~~~, \\
{\rm D}_a B ~&=~ i\,(\gamma^5)_a{}^b\,\psi_b ~~~, \\
{\rm D}_a \psi_b ~&=~ i\,(\gamma^\mu)_{a\,b}~(\partial_\mu A) ~-~ (\gamma^5\gamma^\mu)_{a\,b}~(\partial_\mu B) \\
&\quad~ -~ i\,C_{a\,b}\,F ~+~ (\gamma^5)_{a\,b} G ~~, \\
{\rm D}_a F ~&=~ (\gamma^\mu)_a{}^b~(\partial_\mu\,\psi_b) ~~~, \\
{\rm D}_a G ~&=~ i\,(\gamma^5\gamma^\mu)_a{}^b~(\partial_\mu\,\psi_b) ~~~,
\end{aligned}
$$

and direct calculation shows that

$$
\begin{gathered}
\{~{\rm D}_a\,,\,{\rm D}_b~\}\,A ~=~ i\,2\,(\gamma^\mu)_{a\,b}\,\partial_\mu\,A ~~~,~~~ \{~{\rm D}_a\,,\,{\rm D}_b~\}\,B ~=~ i\,2\,(\gamma^\mu)_{a\,b}\,\partial_\mu\,B ~~~, \\
\{~{\rm D}_a\,,\,{\rm D}_b~\}\,\psi_c ~=~ i\,2\,(\gamma^\mu)_{a\,b}\,\partial_\mu\,\psi_c ~~~, \\
\{~{\rm D}_a\,,\,{\rm D}_b~\}\,F ~=~ i\,2\,(\gamma^\mu)_{a\,b}\,\partial_\mu\,F ~~~,~~~ \{~{\rm D}_a\,,\,{\rm D}_b~\}\,G ~=~ i\,2\,(\gamma^\mu)_{a\,b}\,\partial_\mu\,G ~~~.
\end{gathered}
$$

In four dimensions, the form of the 'super D-equations' are slightly different for the vector supermultiplet

$$
\begin{aligned}
{\rm D}_a\,A_\mu ~&=~ (\gamma_\mu)_a{}^b\,\lambda_b ~~~, \\
{\rm D}_a \lambda_b ~&=~ -~i\,\tfrac{1}{4}([\,\gamma^\mu\,,\,\gamma^\nu\,])_{ab}\,(\,\partial_\mu\,A_\nu ~-~ \partial_\nu\,A_\mu\,) ~+~ (\gamma^5)_{a\,b}\,{\rm d} ~~, \\
{\rm D}_a\,{\rm d} ~&=~ i\,(\gamma^5\gamma^\mu)_a{}^b~\partial_\mu \lambda_b ~~~.
\end{aligned}
$$

and once more direct calculations reveal

$$
\begin{aligned}
\{~{\rm D}_a\,,\,{\rm D}_b~\}\,A_\mu ~&=~ i\,2\,(\gamma^\rho)_{a\,b}\,\partial_\rho\,A_\mu ~-~ \partial_\mu\,r_{a\,b} ~~,~~ r_{a\,b} ~\equiv~ i\,2\,(\gamma^\nu)_{a\,b}\,A_\nu ~~, \\
\{~{\rm D}_a\,,\,{\rm D}_b~\}\,\lambda_c ~&=~ i\,2\,(\gamma^\mu)_{a\,b}\,\partial_\mu\,\lambda_c ~~, \\
\{~{\rm D}_a\,,\,{\rm D}_b~\}\,{\rm d} ~&=~ i\,2\,(\gamma^\mu)_{a\,b}\,\partial_\mu\,{\rm d} ~~.
\end{aligned}
$$

In four dimensions, the form of the 'super D-equations' are slightly different for the tensor supermultiplet

$$
\begin{aligned}
{\rm D}_a \varphi ~&=~ \chi_a ~~~, \\
{\rm D}_a B_{\mu\,\nu} ~&=~ -~\tfrac{1}{4}([\,\gamma_\mu\,,\,\gamma_\nu\,])_a{}^b\,\chi_b ~~~, \\
{\rm D}_a \chi_b ~&=~ i\,(\gamma^\mu)_{a\,b}\,\partial_\mu \varphi ~-~ (\gamma^5\gamma^\mu)_{a\,b}\,\epsilon_\mu{}^{\rho\,\sigma\,\tau}\partial_\rho B_{\sigma\,\tau} ~~.
\end{aligned}
$$

and again direct calculations reveal

$$
\begin{aligned}
\{\, \mathrm{D}_a\,,\, \mathrm{D}_b\,\}\, \varphi &= i\, 2\, (\gamma^\mu)_{a\, b}\, \partial_\mu\, \varphi \quad , \\
\{\, \mathrm{D}_a\,,\, \mathrm{D}_b\,\}\, B_{\mu\, \nu} &= i\, 2\, (\gamma^\rho)_{a\, b}\, \partial_\rho\, B_{\mu\, \nu} ~+~ \partial_\mu\, q_{\nu~a\, b} ~-~ \partial_\nu\, q_{\mu~a\, b} \quad , \\
\{\, \mathrm{D}_a\,,\, \mathrm{D}_b\,\}\, \chi_c &= i\, 2\, (\gamma^\mu)_{a\, b}\, \partial_\mu\, \chi_c \quad , \quad q_{\mu~a\, b} ~\equiv~ i\, 2\, (\gamma^\nu)_{a\, b}\, [\, B_{\mu\, \nu} ~+~ \tfrac{1}{2} \eta_{\mu\, \nu}\, \varphi\,] \quad .
\end{aligned}
$$