From Hypercubes to Adinkra Graphs

The vertices of a cube in a space of any dimension, $\mathcal{N}$, can always be associated with the vertices of a 'vector' of the form

$$(\pm 1, \ \ldots \mathcal{N} - \text{times} \ \ldots \ , \pm 1)$$

so in the example of $\mathcal{N} = 2$, we have the illustration below.

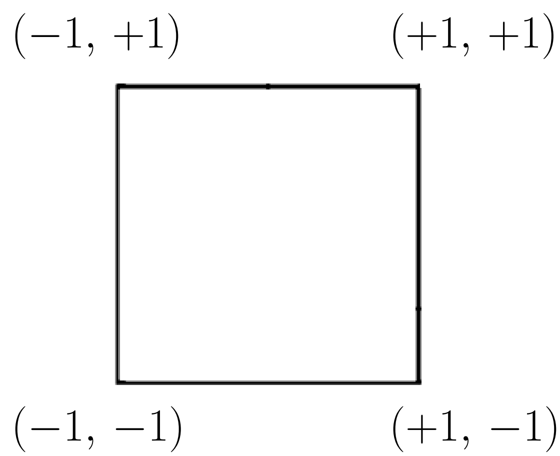


Figure 1

For arbitrary values of $\mathcal{N}$, there are clearly $2^{\mathcal{N}}$ such vertices.

There are several steps required to turn the cube into an adinkra graph.

(1.): Each vertex in the graph must be occupied by either an open node or a closed node. This is done in such a way that as a closed path is traced about any square face of a cubical adinkra, the open and closed nodes appear alternately in the path.

This is illustrated in the Figure 2.

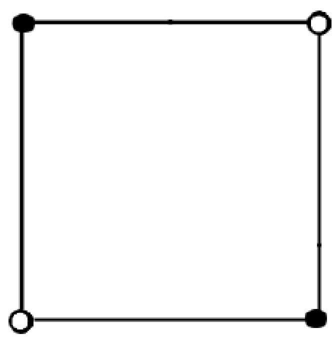


Figure 2

(2.): Each parallel line is given the same color as illustrated in Figure 3.

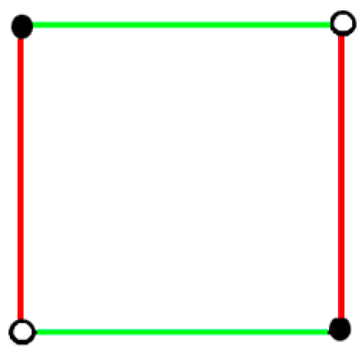


Figure 3

(3.): Every square face must have an odd number of dashed lines as illustrated in Figure 4 below as one particular choice. Since the only odd integers less than four are one or three, one or three of the lines may be dashed. The actual placement of the dashings is irrelevant as any line may be chosen.

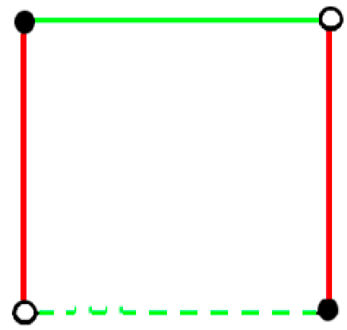


Figure 4

(4.): No open node may appear at the same height as a closed node. For the graph drawn in Figure 4, there are two ways to avoid having this occur. These are shown in Figure 5.

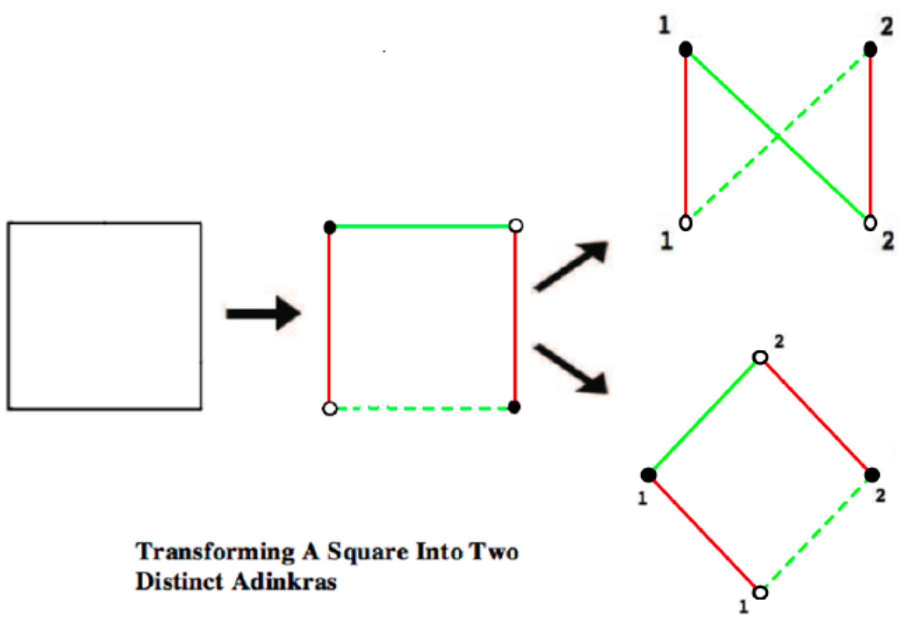


Figure 5

The two adinkras shown to the right of Figure 5 are suggestively called 'the Bow Tie' and 'Diamond' adinkras, respectively.

The ‘decoration process’ described by steps one through four above may be applied to any hypercube no matter what the value of d. These are illustrated briefly by Figure 6 – Figure 8 in the cases of $\mathcal{N} = 3$.

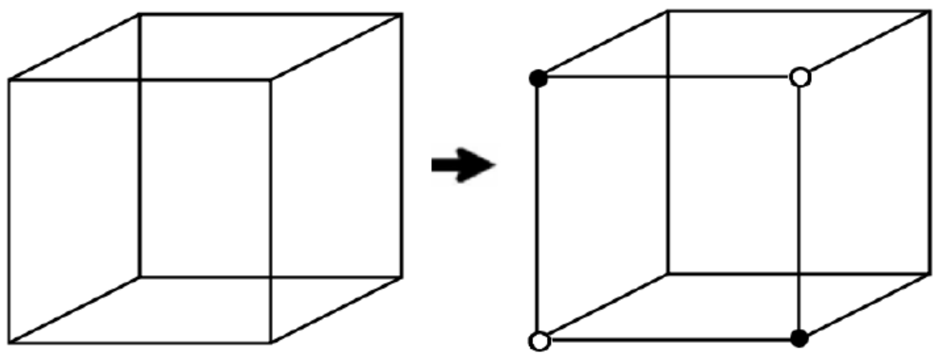


Figure 6

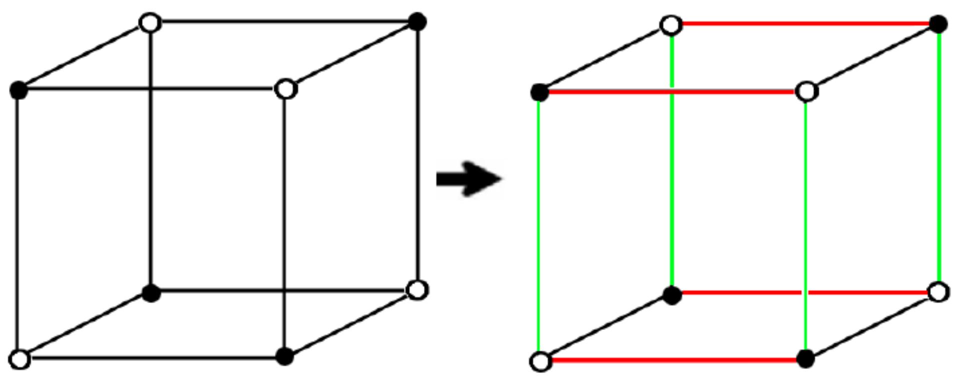


Figure 7

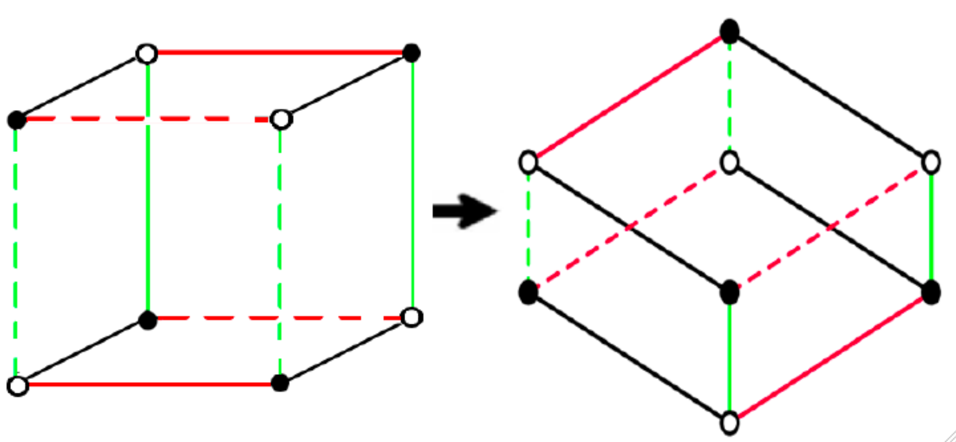
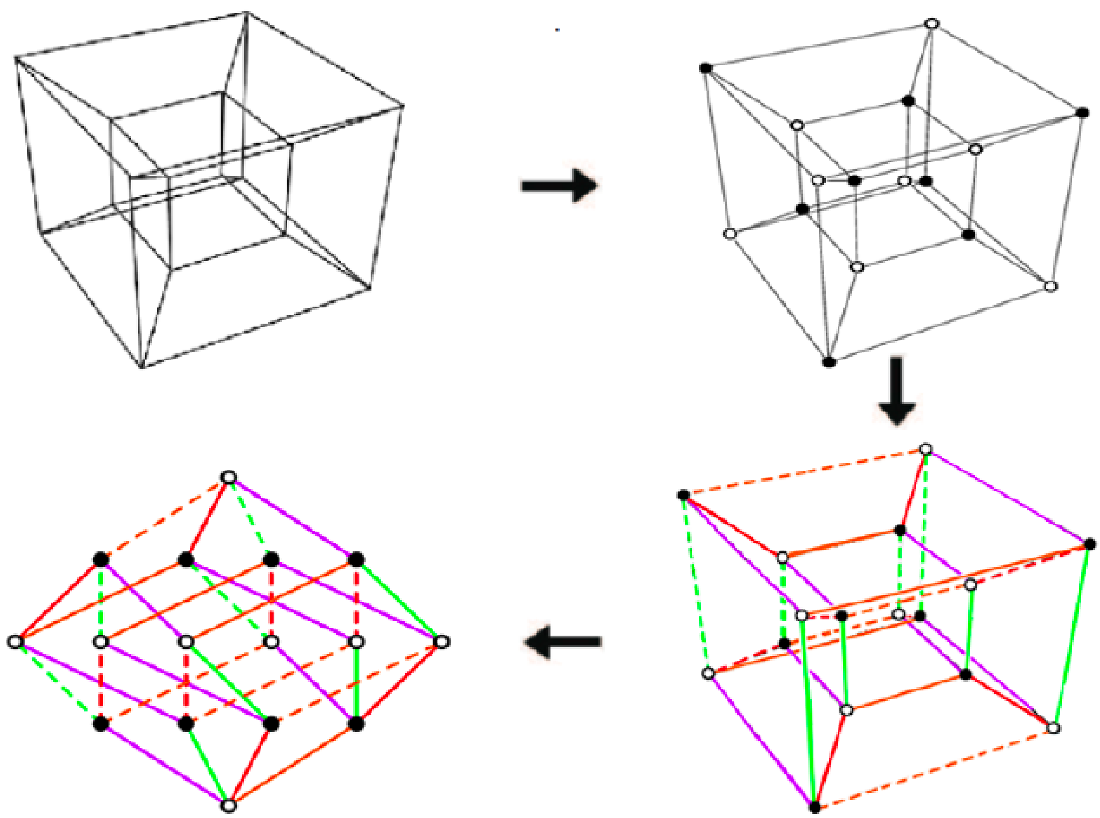


Figure 8

The 'decoration process' described by steps one through four above are illustrated briefly by Figure 9 in the cases of $\mathcal{N} = 4$.



$Figure$ 9

Node Raising & Lowering

The $\mathcal{N} = 2$ cube leads to the Diamond & Bow Tie adinkras 're-drawn' below in Figure 10.

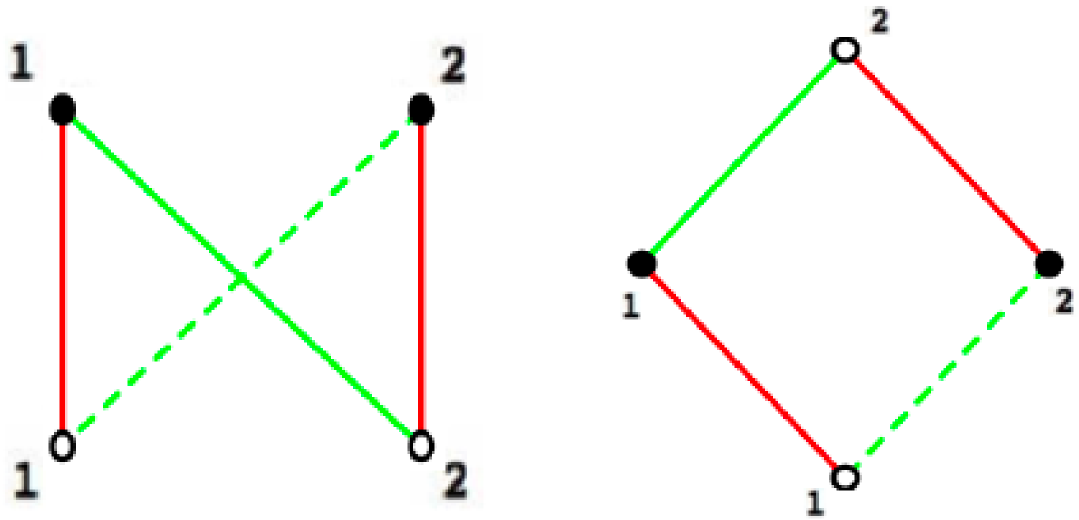


Figure 10

There is obviously a relationship between these two graphs.

If we take the open node denoted by 2 in the Bow Tie adinkra to the left of the figure and raise it to a position above the height of the two closed nodes, the resulting adinkra is equivalent to the Diamond adinkra to the right.

If we take the open node denoted by 2 in the Diamond adinkra to the right of the figure and lower it to a position below the height of the two closed nodes, the resulting adinkra is equivalent to the Bow Tie adinkra to the left.

In figure 11, we have used the Adinkramat to illustrate the node lowering process on the adinkra associated with the tesseract.

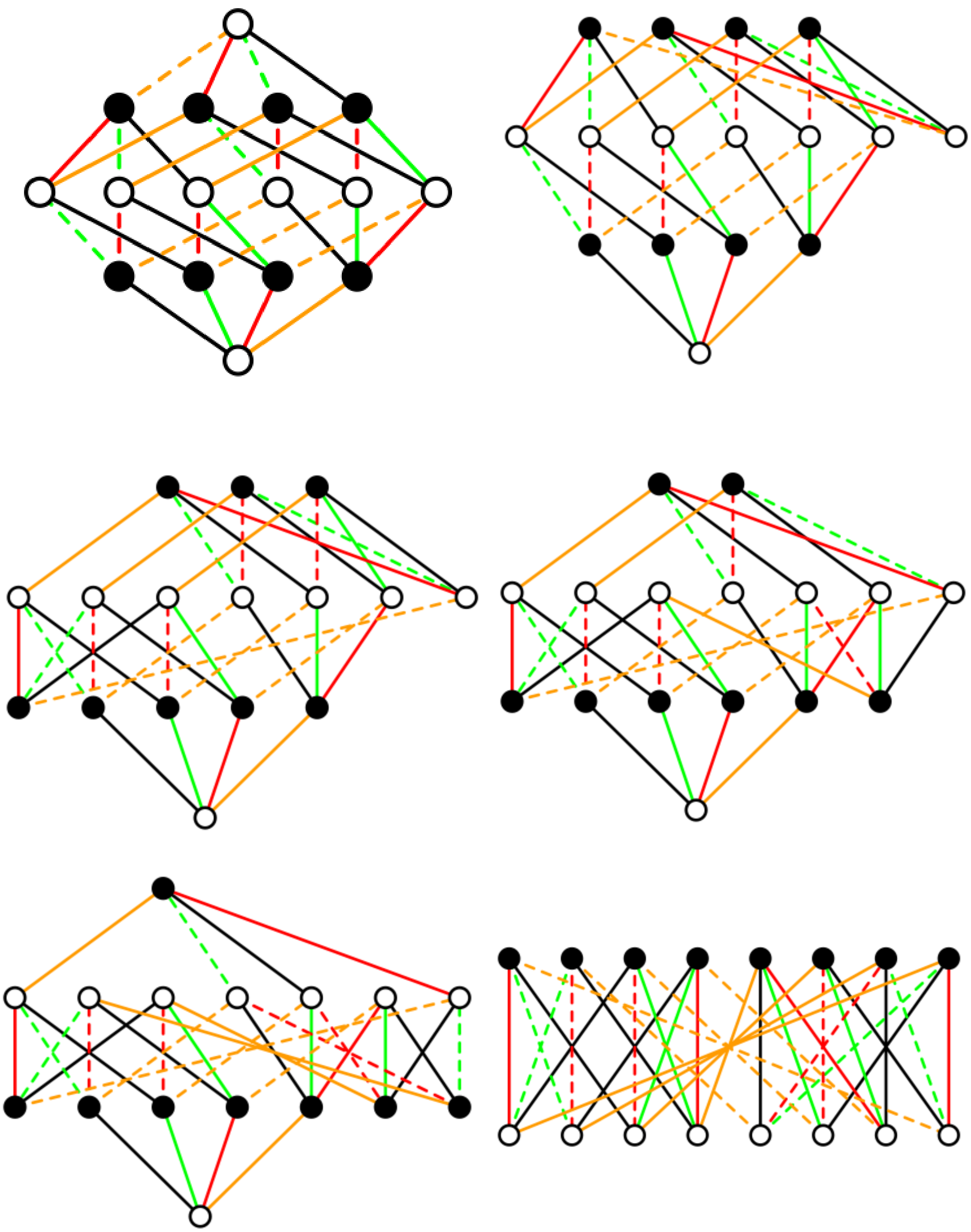


Figure 11

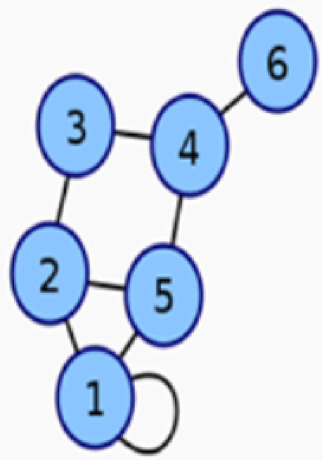
The $\mathcal{N} = 2$ cube leads to Bow Tie graph seen in the upper section of the right hand portion of Figure 5.

The first adinkra in the uppermost left corner of Figure 11 is the analog of the Diamond and the final adinkra in the lowermost right hand corner is the analog of the Bow Tie. The first of these is called a 'one-hooked' adinkra. The latter is called a 'valise' adinkra. A valise adinkra is one where all the open nodes appear at the same height in the diagram and all the closed notes have the same height but one that is distinct from that of the open nodes. The other adinkras shown are simply a selection of intermediate adinkras that can be constructed by lowering successive nodes of the initial one-hooked adinkra.

The Adinkra/Adjacency Matrix Relationship

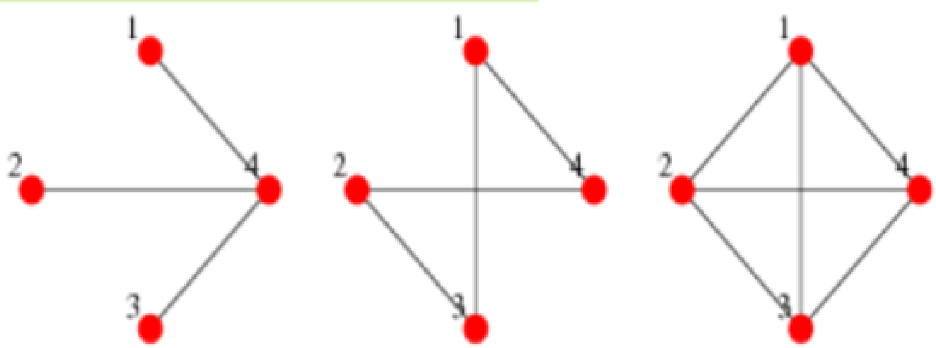
http://en.wikipedia.org/wiki/Adjacency_matrix

Adjacency matrix

Labeled graph	Adjacency matrix
	$\begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$ Coordinates are 1-6.

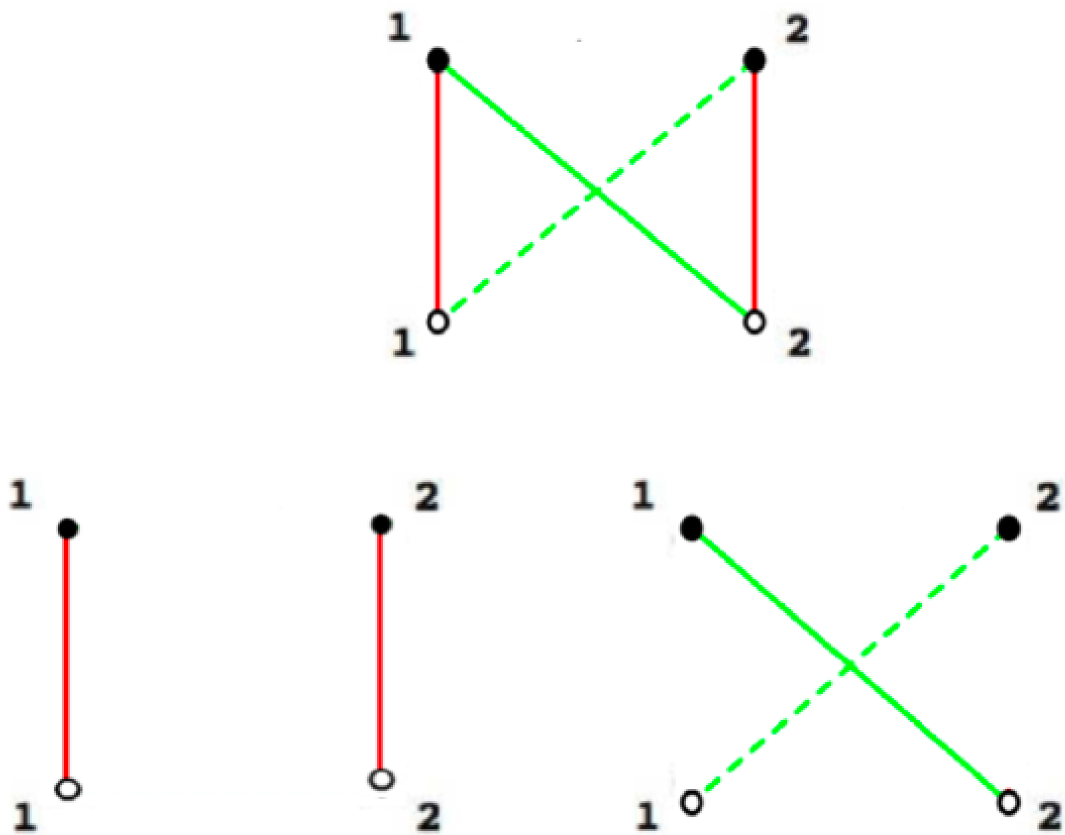
http://mathworld.wolfram.com/AdjacencyMatrix.html

Adjacency Matrix



$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

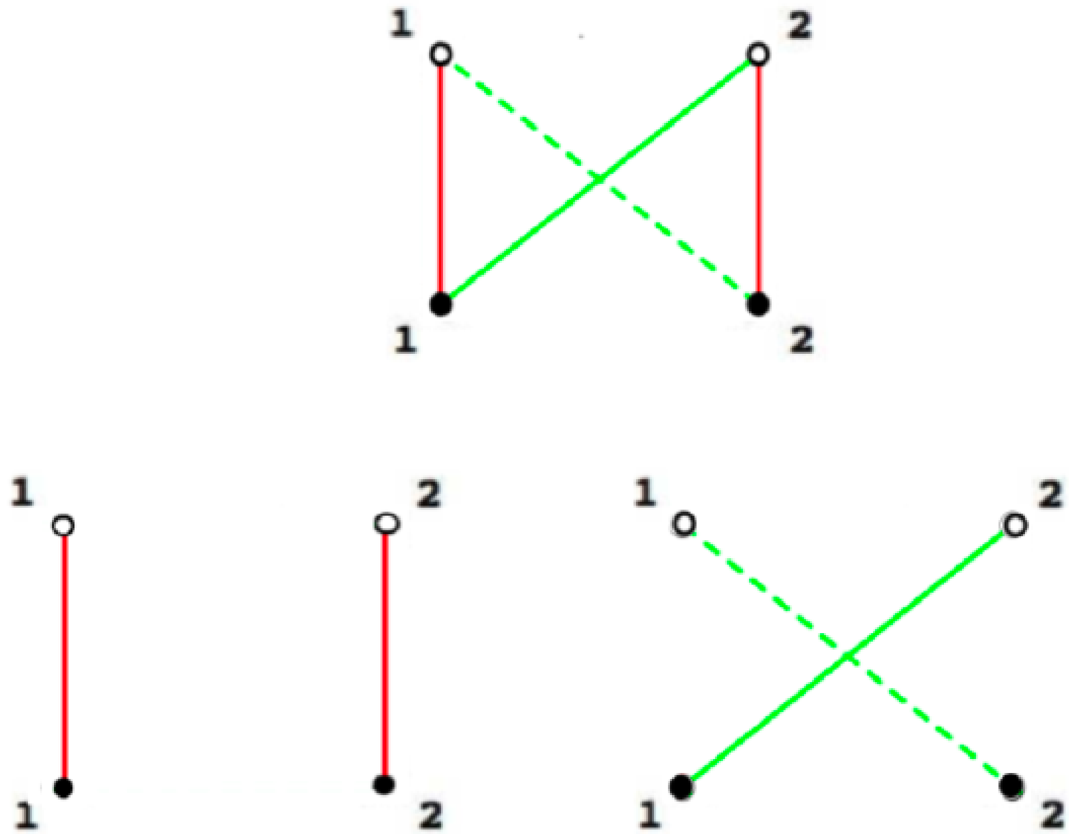
From Adinkra Graphs to Matrices



L − Matrices

$$L_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad L_2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

Figure 1

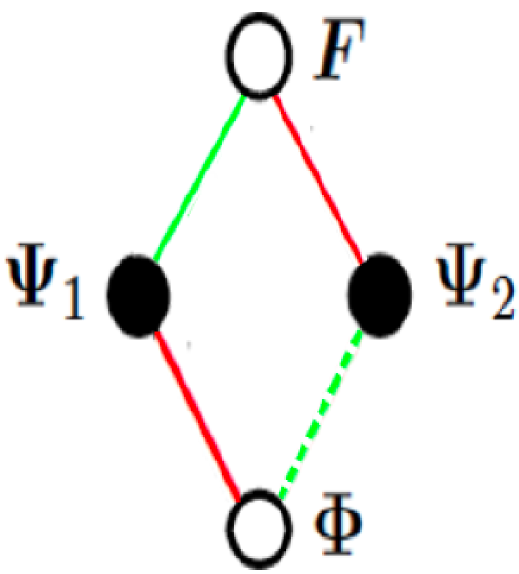


R – Matrices

$$R_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad R_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Figure 2

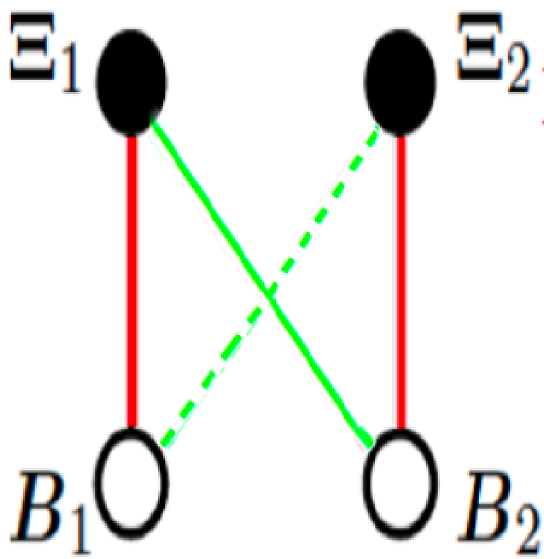
From Adinkra Graphs to SUSY 1D Representation



$$\mathrm{D}_1 \Phi = i\,\Psi_1 \,, \quad \mathrm{D}_2 \Phi = -i\,\Psi_2 \,,$$
$$\mathrm{D}_1 \Psi_1 = \dot{\Phi} \,, \quad \mathrm{D}_2 \Psi_1 = F \,,$$
$$\mathrm{D}_1 \Psi_2 = F \,, \quad \mathrm{D}_2 \Psi_2 = -\,\dot{\Phi} \,,$$
$$\mathrm{D}_1 F = i\dot{\Psi}_2 \,, \quad \mathrm{D}_2 F = i\,\dot{\Psi}_1 \,.$$

Color indicates direction,
Height indicates engineering dimension,
Solid lines indicate positive coefficients,
Dashed lines indicate negative coefficients.

Figure 3



$$
\begin{aligned}
\mathrm{D}_1 B_1 &= i\,\Xi_1\,, & \mathrm{D}_2 B_1 &= -i\,\Xi_2\,,\\
\mathrm{D}_1 B_2 &= i\,\Xi_2\,, & \mathrm{D}_2 B_2 &= i\,\Xi_1\,,\\
\mathrm{D}_1 \Xi_1 &= \dot{B}_1\,, & \mathrm{D}_2 \Xi_1 &= \dot{B}_2\,,\\
\mathrm{D}_1 \Xi_2 &= \dot{B}_2\,, & \mathrm{D}_2 \Xi_2 &= -\dot{B}_1\,.
\end{aligned}
$$

Figure 4

The Valise Supermultiplet Structure

The Bosonic Ring

$$\Phi_i(t_1)\,\Phi_j(t_2) \;=\; +\;\Phi_j(t_2)\,\Phi_i(t_1)$$

The Fermionic Ring

$$\Psi_{\hat{i}}(t_1)\,\Psi_{\hat{j}}(t_2) \;=\; -\;\Psi_{\hat{j}}(t_2)\,\Psi_{\hat{i}}(t_1)$$

From yesterday's lecture, we came to understand that there are a large number of adinkra graphs that can be used to specific how the abstract supercharge operator is realized in a set of equations that involve bosons and fermions.

Valise Formulation

$$\mathrm{D_I}\Phi_i \;=\; i\,(\mathrm{L_I})_{i\,\hat{j}}\,\Psi_{\hat{j}} \qquad ,$$

$$\mathrm{D_I}\Psi_{\hat{k}} \;=\; (\mathrm{R_I})_{\hat{k}\,j}\,\frac{d}{dt}\,\Phi_j \qquad .$$

Codes: The Resolution of a Puzzle

With some work, it can be shown that the following three adinkras satisfy the algebra of SUSY with $\mathcal{N} = 4$.

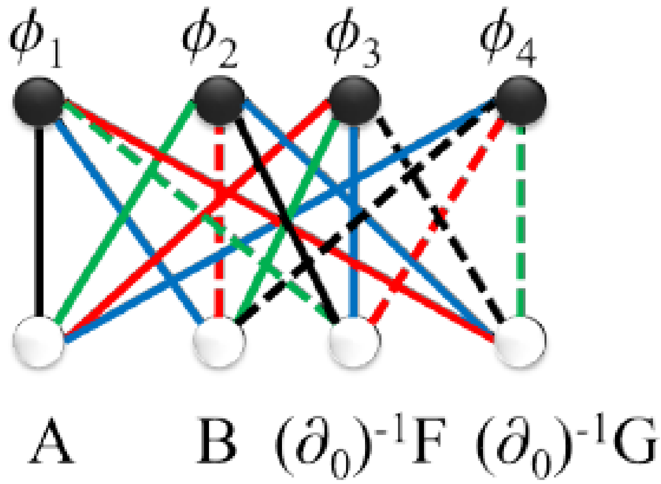


Figure 5

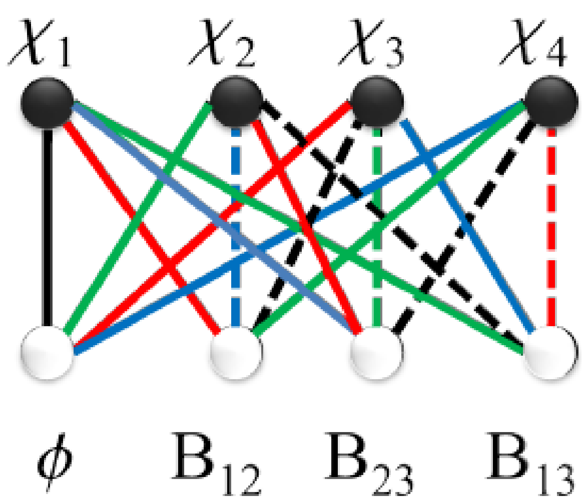


Figure 6

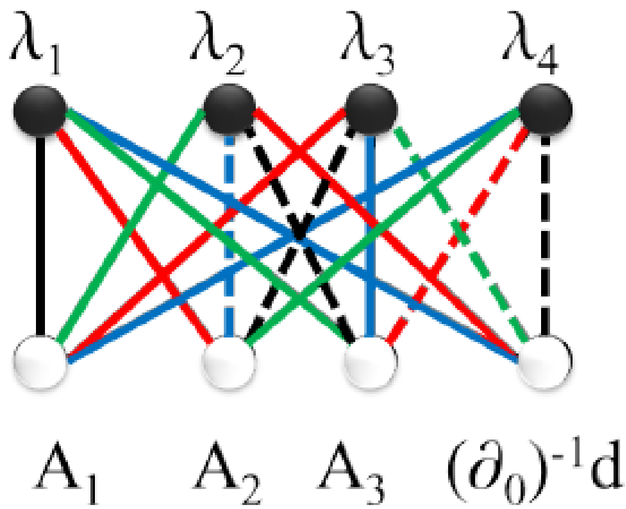


Figure 7

Q: Since $\mathcal{N} = 4$, there should be how many open nodes associated with such adinkras?

Q: Since $\mathcal{N} = 4$, there should be how many closed nodes associated with such adinkras?

A: Since $2^{\mathcal{N}} = 16$, there should be eight open nodes and eight closed nodes associated with such adinkras?

Conclusion: Such adinkras cannot be associated with the tesseract!

Revisiting The Tesseract

	K	ζ_1	ζ_2	ζ_3	ζ_4	X_{12}	X_{13}	X_{14}	X_{23}	X_{24}	X_{34}	$-\Lambda_2$	Λ_1	Λ_4	$-\Lambda_3$	d
D_1:	ζ_1	$i\dot{K}$	iX_{12}	iX_{13}	iX_{14}	$\dot{\zeta}_2$	$\dot{\zeta}_3$	$\dot{\zeta}_4$	$-\Lambda_3$	$-\Lambda_4$	Λ_1	id	$i\dot{X}_{34}$	$-i\dot{X}_{24}$	$i\dot{X}_{23}$	$-\dot{\Lambda}_2$
D_2:	ζ_2	$-iX_{12}$	$i\dot{K}$	iX_{23}	iX_{24}	$-\dot{\zeta}_1$	Λ_3	Λ_4	$\dot{\zeta}_3$	$\dot{\zeta}_4$	Λ_2	$-i\dot{X}_{34}$	id	$i\dot{X}_{14}$	$-i\dot{X}_{13}$	$\dot{\Lambda}_1$
D_3:	ζ_3	$-iX_{13}$	$-iX_{23}$	$i\dot{K}$	iX_{34}	$-\Lambda_3$	$-\dot{\zeta}_1$	$-\Lambda_1$	$-\dot{\zeta}_2$	$-\Lambda_2$	$\dot{\zeta}_4$	$i\dot{X}_{24}$	$-i\dot{X}_{14}$	id	$i\dot{X}_{12}$	$\dot{\Lambda}_4$
D_4:	ζ_4	$-iX_{14}$	$-iX_{24}$	$-iX_{34}$	$i\dot{K}$	$-\Lambda_4$	Λ_1	$-\dot{\zeta}_1$	Λ_2	$-\dot{\zeta}_2$	$-\dot{\zeta}_3$	$-i\dot{X}_{23}$	$i\dot{X}_{13}$	$-i\dot{X}_{12}$	id	$-\dot{\Lambda}_3$

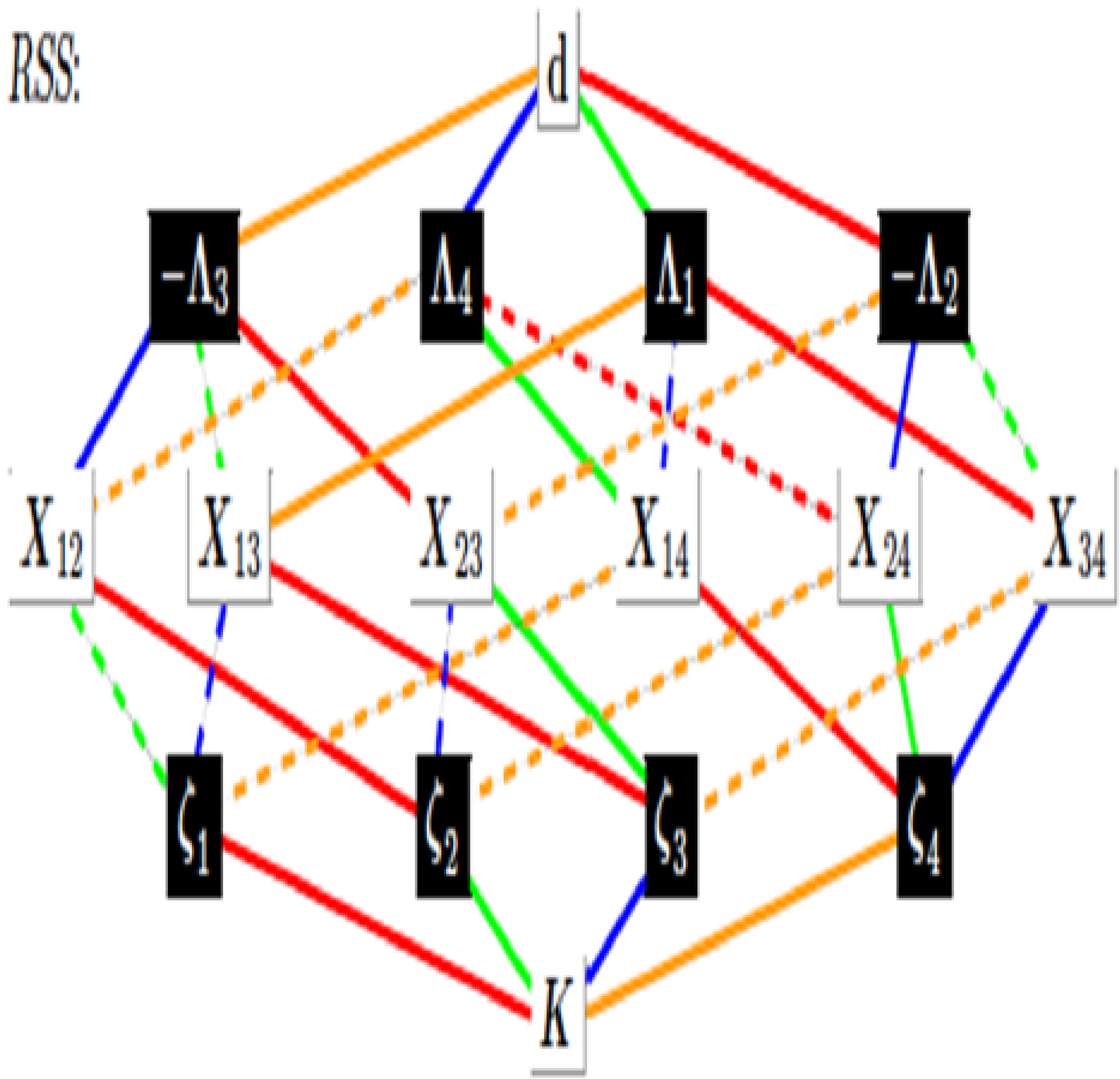


Figure 8

Bits Naturally Arise From The Geometry Of Hypercubes

To our knowledge, this situation marks the first time equations of fundamental physics point toward the relevance of SDEC's, the most famous being the Hamming Code.

In view of the 'It From Bit' hypothesis of John Wheeler, one has to wonder about the possibility of a larger previously unseen role for information theory.

Feynman on Wheeler

Feynman, Wheeler's student in the 1940s, turned to Thorne, Wheeler's student in the 1960s, and said,

"This guy sounds crazy. What people of your generation don't know is that he has always sounded crazy. But when I was his student, I discovered that if you take one of his crazy ideas and you unwrap the layers of craziness from it one after another like lifting the layers off an onion, at the heart of the idea you will often find a powerful kernel of truth."

Bits Naturally Arise From The Geometry Of Hypercubes

Bits naturally appear in any situation where cubical geometry is relevant. The vertices of a cube can always be written in the form

$$(\ \pm 1,\ \pm 1,\ \pm 1,\ \ldots,\ \pm 1\ )$$

or re-written in the form

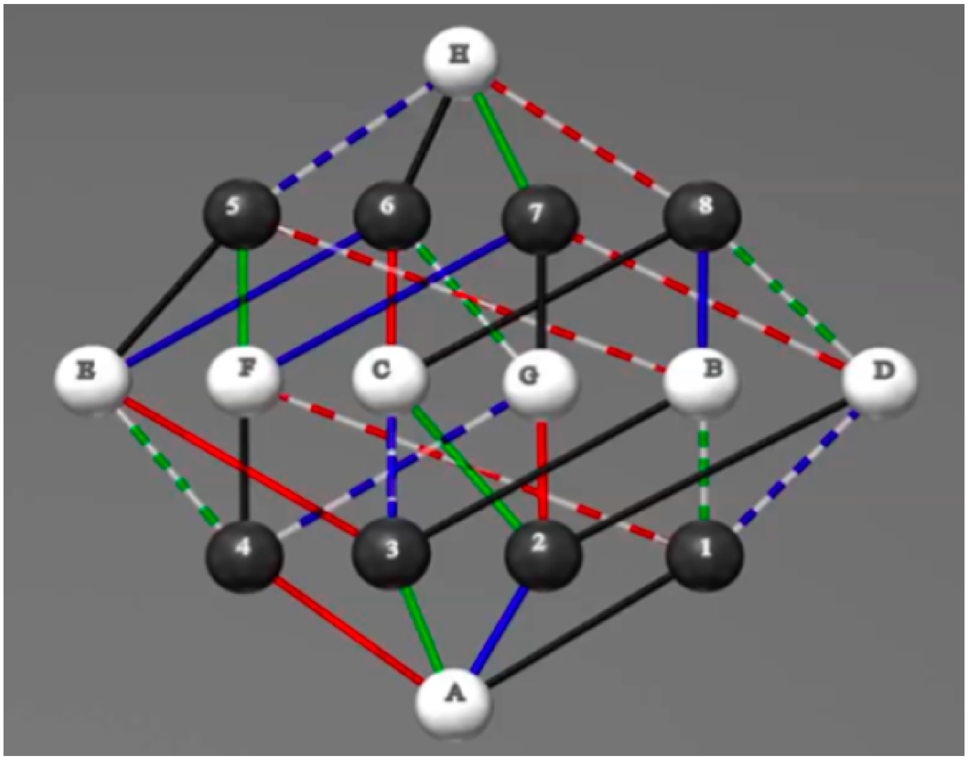
$$(\ (-1)^{p_1},\ (-1)^{p_2},\ (-1)^{p_3},\ \ldots,\ (-1)^{p_d}\ )$$

where the exponents are bits since they take on values 1 or 0. Thus any vertex has an 'address' that is a string of bits

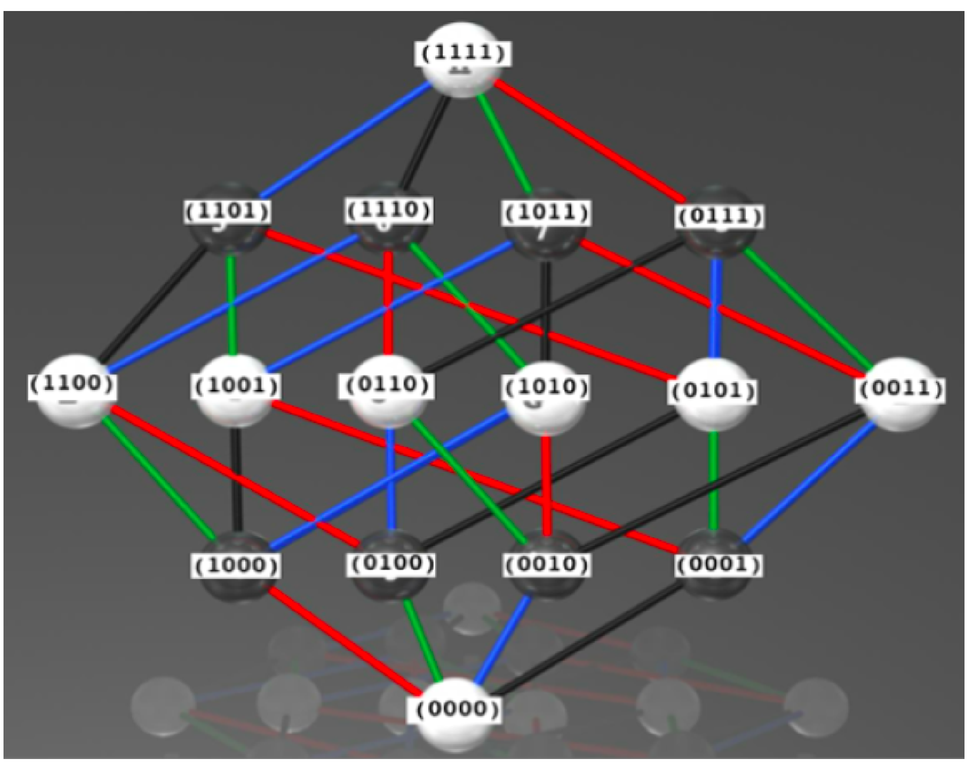
$$(\ \boldsymbol{p}_1,\ \boldsymbol{p}_2,\ \boldsymbol{p}_3,\ \ldots,\ \boldsymbol{p}_d\ )$$

the information theoretic definition of a 'word.'

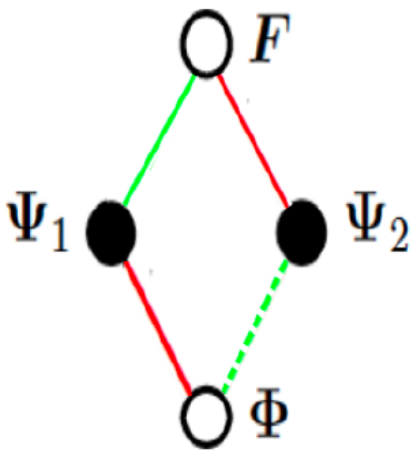
Below is illustrated an adinkra built on the basis of a tesseract.



Below is illustrated the same adinkra without the dashing but including the information-theoretic bit-word address of each node.



What Happens When Codes Are Not Used?



$$\mathrm{D}_1 \Phi = i\, \Psi_1 \,, \quad \mathrm{D}_2 \Phi = - i\, \Psi_2 \,,$$
$$\mathrm{D}_1 \Psi_1 = \dot{\Phi} \,, \quad \mathrm{D}_2 \Psi_1 = F \,,$$
$$\mathrm{D}_1 \Psi_2 = F \,, \quad \mathrm{D}_2 \Psi_2 = - \dot{\Phi} \,,$$
$$\mathrm{D}_1 F = i \dot{\Psi}_2 \,, \quad \mathrm{D}_2 F = i\, \dot{\Psi}_1 \,.$$

Color indicates direction,
Height indicates engineering dimension,
Solid lines indicate positive coefficients,
Dashed lines indicate negative coefficients.

Let us try to identify the uppermost node by letting it be linearly depend on the lowest node.

$$F \;=\; \kappa \, \Phi$$

This has a number of implications.

$$\begin{aligned} \mathrm{D}_1 \; F \;&=\; \kappa \, \mathrm{D}_1 \Phi \\ i \, \dot{\Psi}_2 \;&=\; i \, \kappa \, \Psi_1 \end{aligned}$$

$$\begin{aligned} \mathrm{D}_2 \; F \;&=\; \kappa \, \mathrm{D}_2 \Phi \\ i \, \dot{\Psi}_1 \;&=\; - \, i \, \kappa \, \Psi_2 \end{aligned}$$

$$\frac{d^2}{dt^2}\Psi_1 \;=\; -\,\kappa^2\,\Psi_1 \quad , \quad \frac{d^2}{dt^2}\Psi_2 \;=\; -\,\kappa^2\,\Psi_2 \quad ,$$

Since we also have the two equations

$$\dot{\Psi}_2 \;=\; \kappa\,\Psi_1 \quad , \quad \dot{\Psi}_1 \;=\; -\kappa\,\Psi_2$$

We can use these to derive more information.

$$\mathrm{D}_2\dot{\Psi}_2 \;=\; \kappa\,\mathrm{D}_2\Psi_1$$

$$-\;\frac{d^2}{dt^2}\Phi \;=\; \kappa\,\mathrm{D}_2\Psi_1$$

$$\frac{d^2}{dt^2}\Phi \;=\; -\,\kappa\,\mathrm{D}_2\Psi_1$$

$$\frac{d^2}{dt^2}\Phi \;=\; -\,\kappa\,F$$

$$\frac{d^2}{dt^2}\Phi \;=\; -\,\kappa^2\,\Phi$$

So the condition

$$F \;=\; \kappa\,\Phi$$

implies

(1.)

$$\frac{d^2}{dt^2}\Phi \;=\; -\,\kappa^2\,\Phi \qquad ,$$

(2.)

$$\frac{d^2}{dt^2}\Psi_1 \;=\; -\,\kappa^2\,\Psi_1 \qquad ,$$

(3.)

$$\frac{d^2}{dt^2}\Psi_2 \;=\; -\,\kappa^2\,\Psi_2 \qquad .$$

The Broken Word Problem

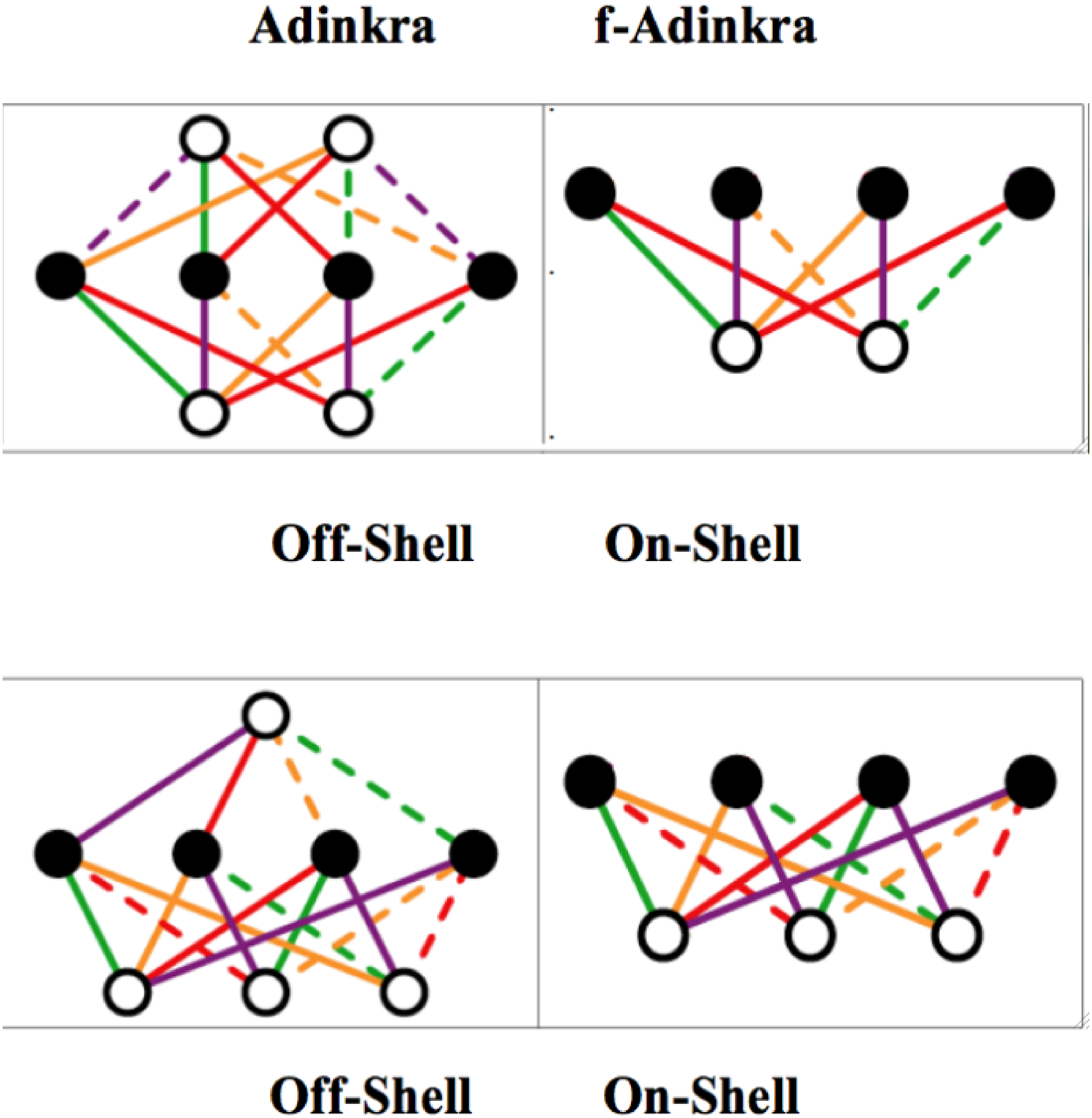


Figure 1

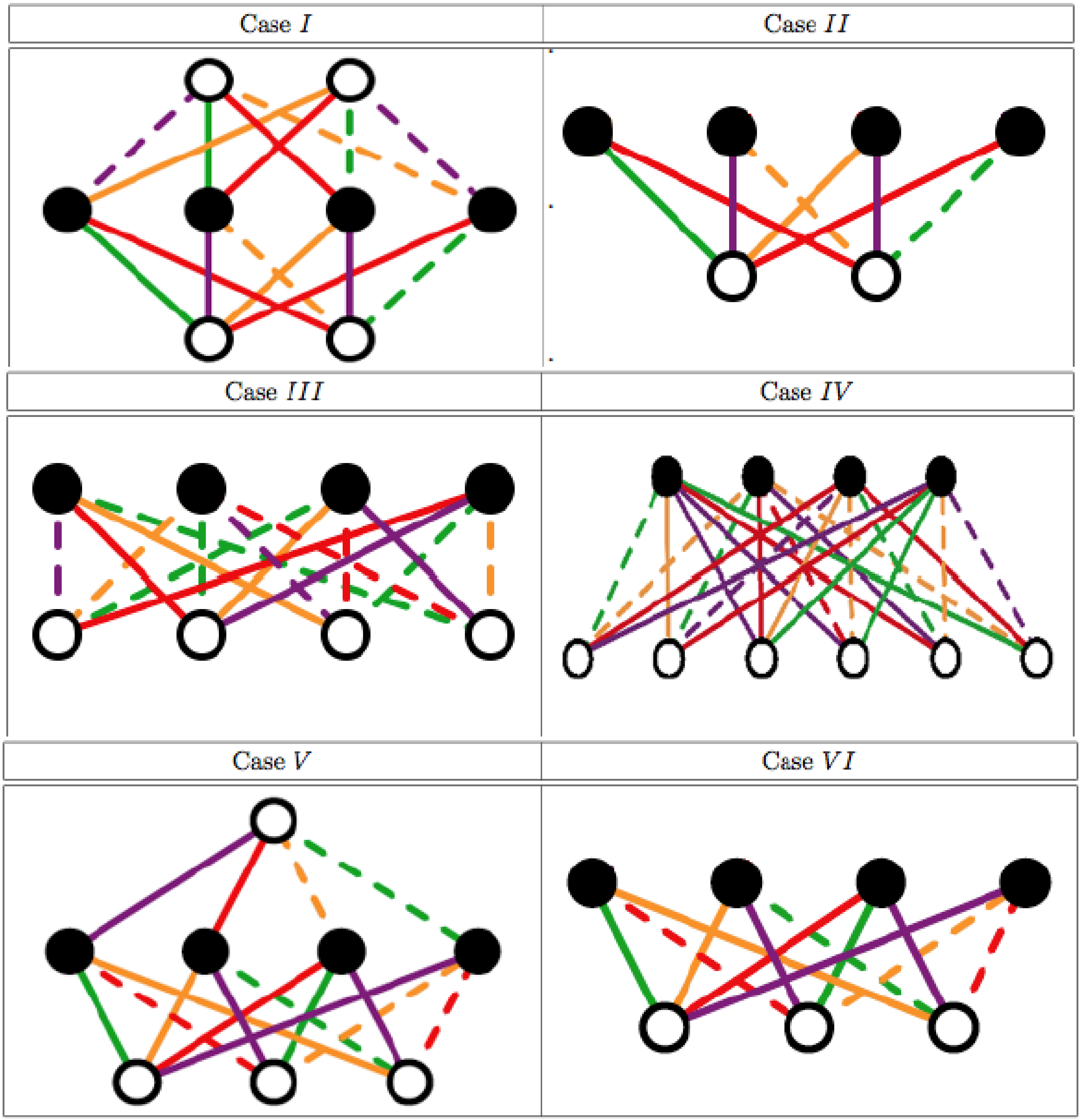


Figure 2

Is it possible to add nodes and links to the graph below so that the total extension becomes an adinkra?

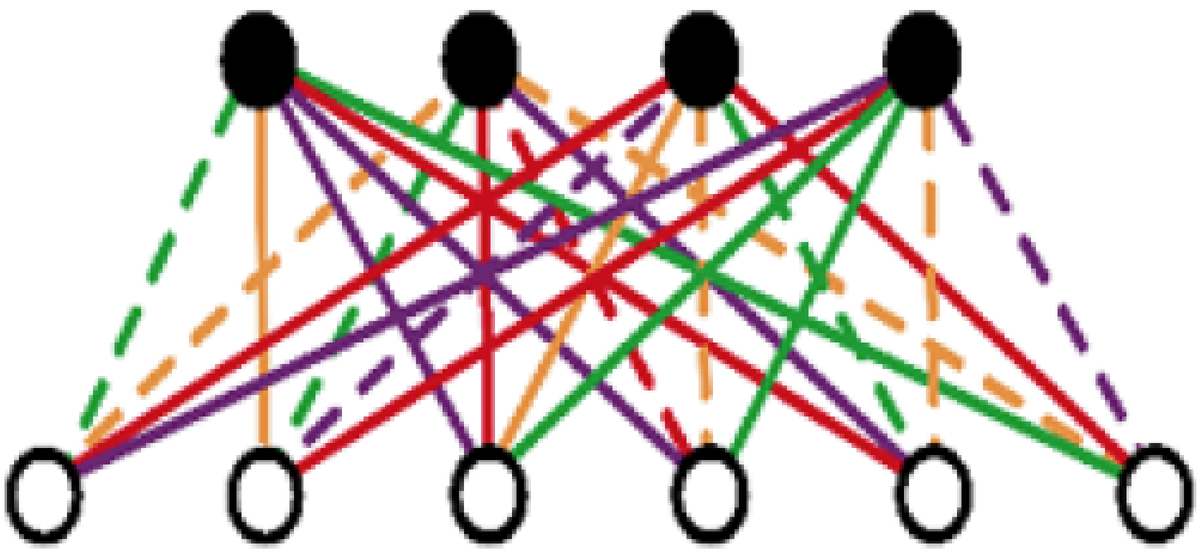


Figure 3

This is a version of the 'off-shell' or 'auxiliary field' problem that has remained unsolved in over thirty years.