

From Hypercubes to Adinkra Graphs

The vertices of a cube in a space of any dimension, \mathcal{N} , can always be associated with the vertices of a ‘vector’ of the form

$$(\pm 1, \dots \mathcal{N} - \text{times} \dots, \pm 1)$$

so in the example of $\mathcal{N} = 2$, we have the illustration below.

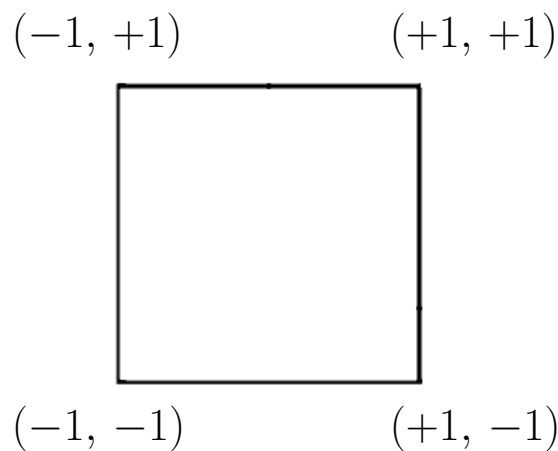


Figure 1

For arbitrary values of \mathcal{N} , there are clearly $2^{\mathcal{N}}$ such vertices.

There are several steps required to turn the cube into an adinkra graph.

- (1.): Each vertex in the graph must be occupied by either an open node or a closed node. This is done in such a way that as a closed path is traced about any square face of a cubical adinkra, the open and closed nodes appear alternately in the path.

This is illustrated in the Figure 2.

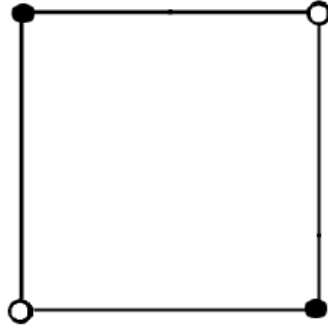


Figure 2

(2.): Each parallel line is given the same color as illustrated in Figure 3.

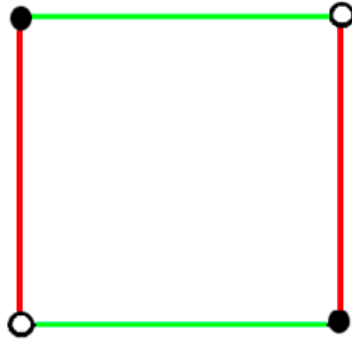


Figure 3

(3.): Every square face must have an odd number of dashed lines as illustrated in Figure 4 below as one particular choice. Since the only odd integers less than four are one or three, one or three of the lines may be dashed. The actual placement of the dashings is irrelevant as any line may be chosen.

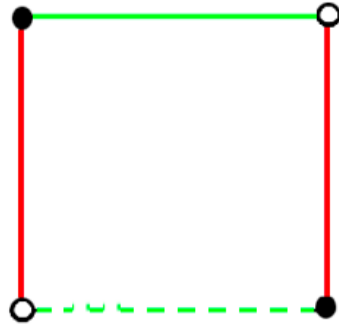


Figure 4

(4.): No open node may appear at the same height as a closed node. For the graph drawn in Figure 4, there are two ways to avoid having this occur. These are shown in Figure 5.

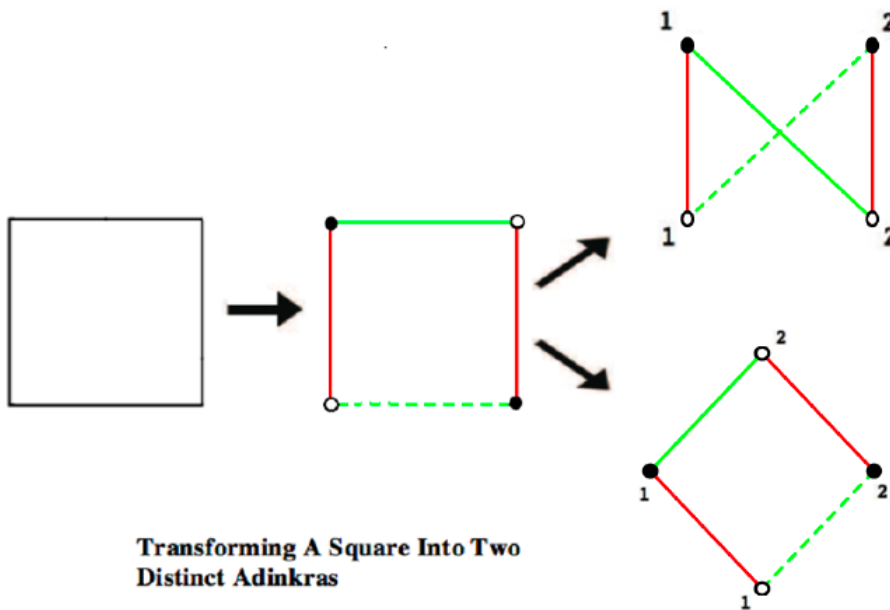


Figure 5

The two adinkras shown to the right of Figure 5 are suggestively called ‘the Bow Tie’ and ‘Diamond’ adinkras, respectively.

The ‘decoration process’ described by steps one through four above may be applied to any hypercube no matter what the value of d . These are illustrated briefly by Figure 6 – Figure 8 in the cases of $\mathcal{N} = 3$.

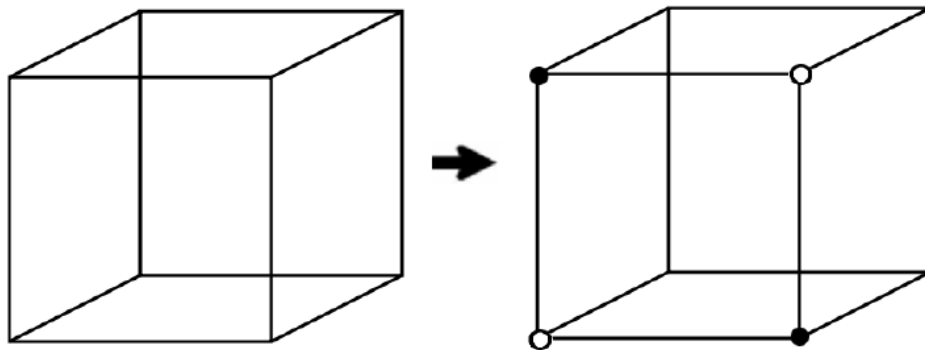


Figure 6

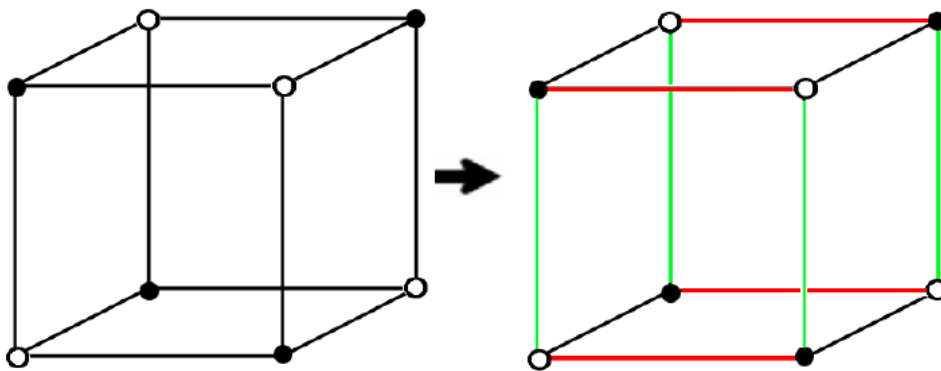


Figure 7

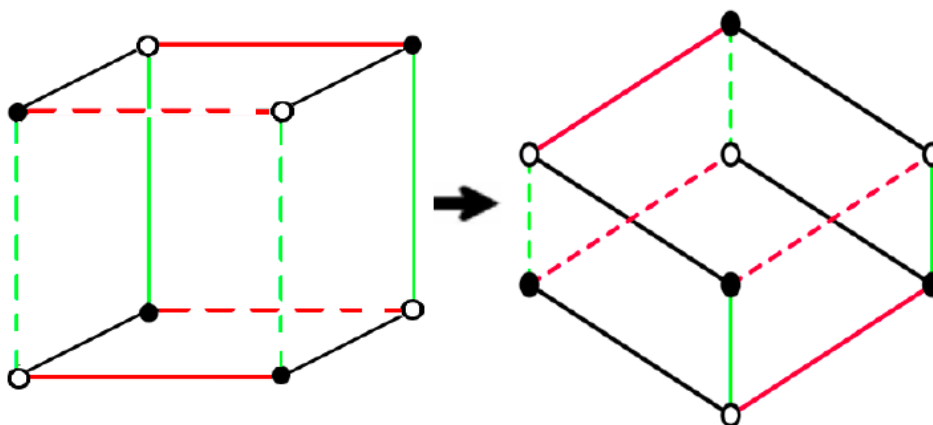


Figure 8

The 'decoration process' described by steps one through four above are illustrated briefly by Figure 9 in the cases of $\mathcal{N} = 4$.

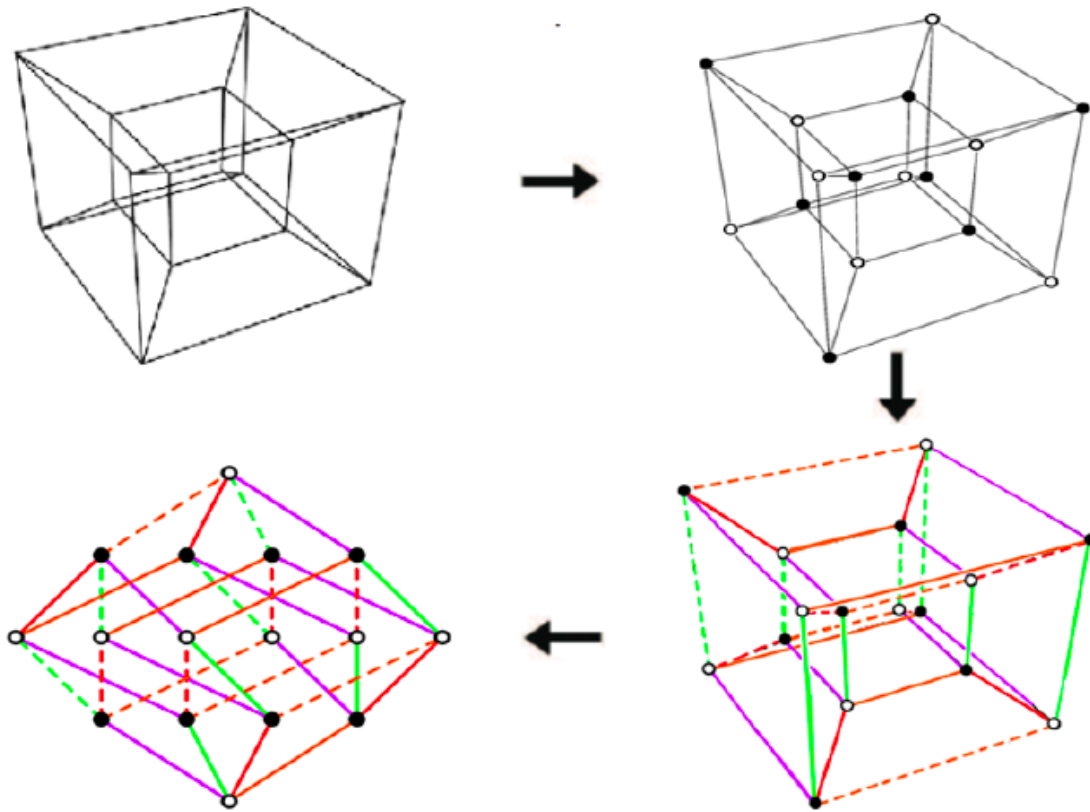


Figure 9

Node Raising & Lowering

The $\mathcal{N} = 2$ cube leads to the Diamond & Bow Tie adinkras 're-drawn' below in Figure 10.

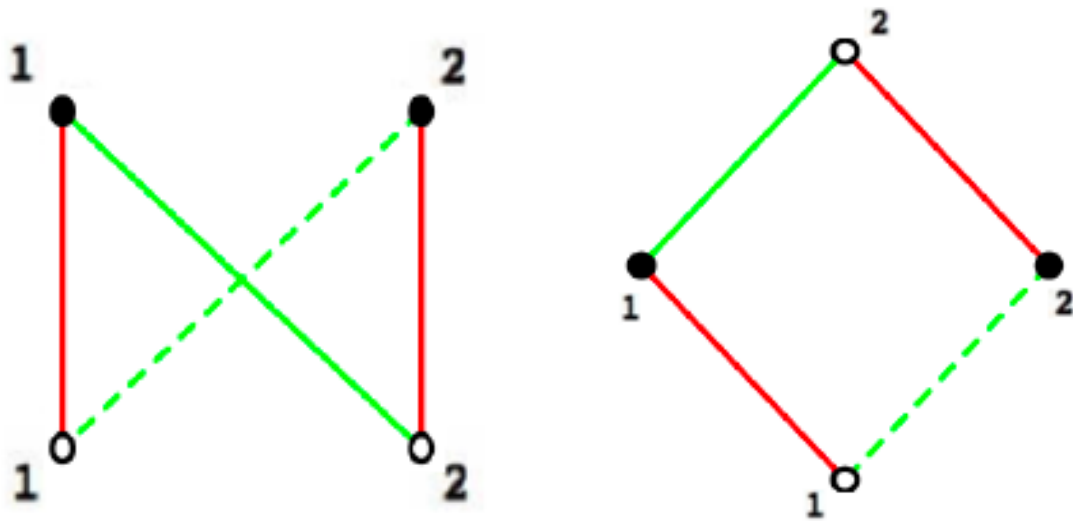


Figure 10

There is obviously a relationship between these two graphs.

If we take the open node denoted by 2 in the Bow Tie adinkra to the left of the figure and raise it to a position above the height of the two closed nodes, the resulting adinkra is equivalent to the Diamond adinkra to the right.

If we take the open node denoted by 2 in the Diamond adinkra to the right of the figure and lower it to a position below the height of the two closed nodes, the resulting adinkra is equivalent to the Bow Tie adinkra to the left.

In figure 11, we have used the Adinkramat to illustrate the node lowering process on the adinkra associated with the tesseract.

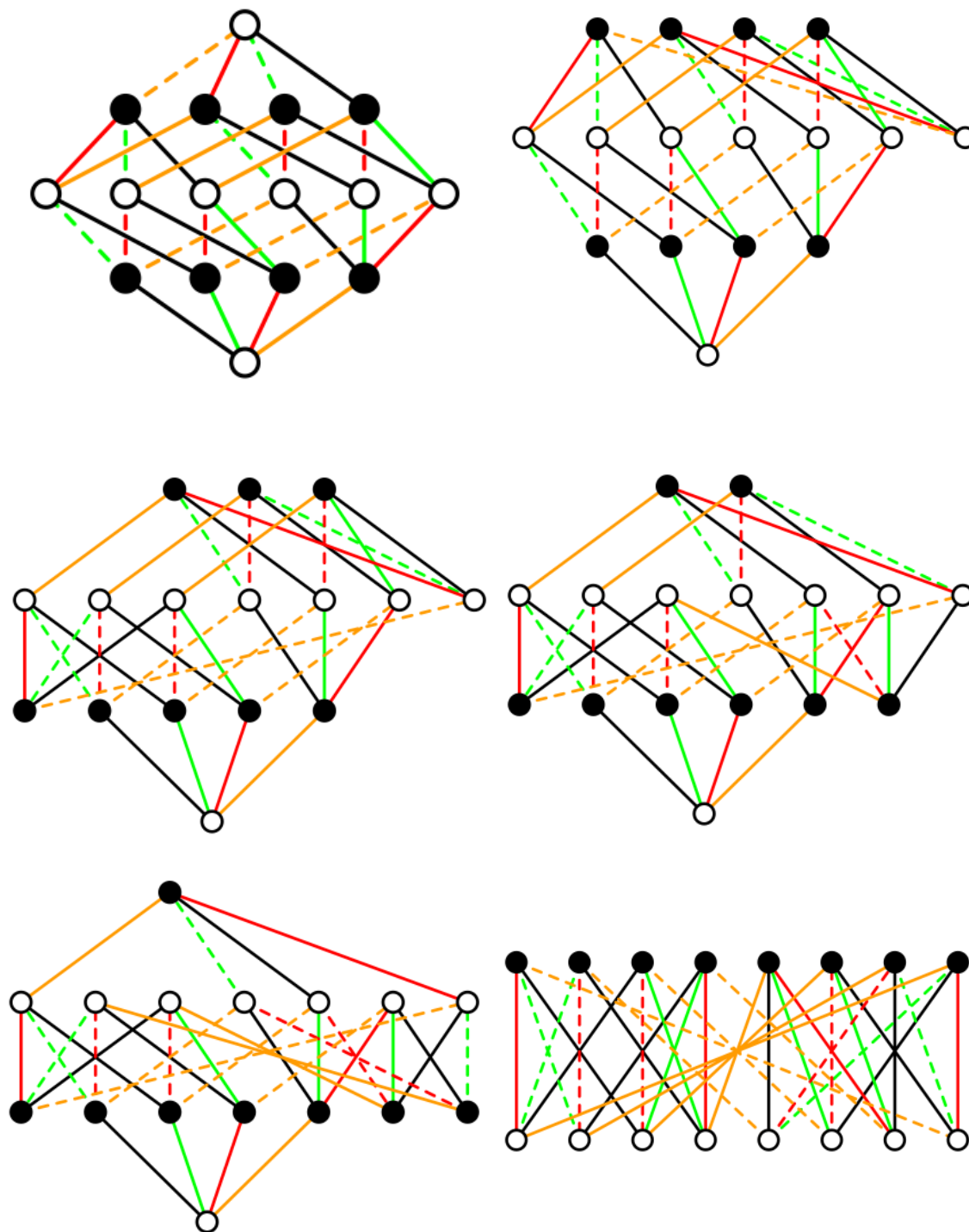


Figure 11

The $\mathcal{N} = 2$ cube leads to Bow Tie graph seen in the upper section of the right hand portion of Figure 5.

The first adinkra in the uppermost left corner of Figure 11 is the analog of the Diamond and the final adinkra in the lowermost right hand corner is the analog of the Bow Tie. The first of these is called a ‘one-hooked’ adinkra. The latter is called a ‘valise’ adinkra. A valise adinkra is one where all the open nodes appear at the same height in the diagram and all the closed nodes have the same height but one that is distinct from that of the open nodes. The other adinkras shown are simply a selection of intermediate adinkras that can be constructed by lowering successive nodes of the initial one-hooked adinkra.