4D Chiral Supermultiplet

$(\boldsymbol{A},\ \boldsymbol{B},\ \boldsymbol{\psi_a},\ \boldsymbol{F},\ \boldsymbol{G})$

$$\mathrm{D}_a A = \psi_a \ ,$$

$$\mathrm{D}_a B = i\,(\gamma^5)_a{}^b\,\psi_b \ ,$$

$$\mathrm{D}_a \psi_b = i\,(\gamma^\mu)_{ab}\,\partial_\mu A \,-\, (\gamma^5\gamma^\mu)_{ab}\,\partial_\mu B \,-\, i\,C_{ab}\,F \,+\, (\gamma^5)_{ab}\,G \ ,$$

$$\mathrm{D}_a F = (\gamma^\mu)_a{}^b\,\partial_\mu\,\psi_b \ ,$$

$$\mathrm{D}_a G = i\,(\gamma^5\gamma^\mu)_a{}^b\,\partial_\mu\,\psi_b \ .$$

4D Vector Supermultiplet

$(\boldsymbol{A_\mu}, \boldsymbol{\lambda_a}, \mathbf{d})$

$$\mathrm{D}_a A_\mu = (\gamma_\mu)_a{}^b \lambda_b \quad ,$$

$$\mathrm{D}_a \lambda_b = - i \tfrac{1}{4} ([\gamma^\mu, \gamma^\nu])_{ab} (\partial_\mu A_\nu - \partial_\nu A_\mu) + (\gamma^5)_{ab} \mathrm{d} \ ,$$

$$\mathrm{D}_a \mathrm{d} = i (\gamma^5 \gamma^\mu)_a{}^b \partial_\mu \lambda_b \quad .$$

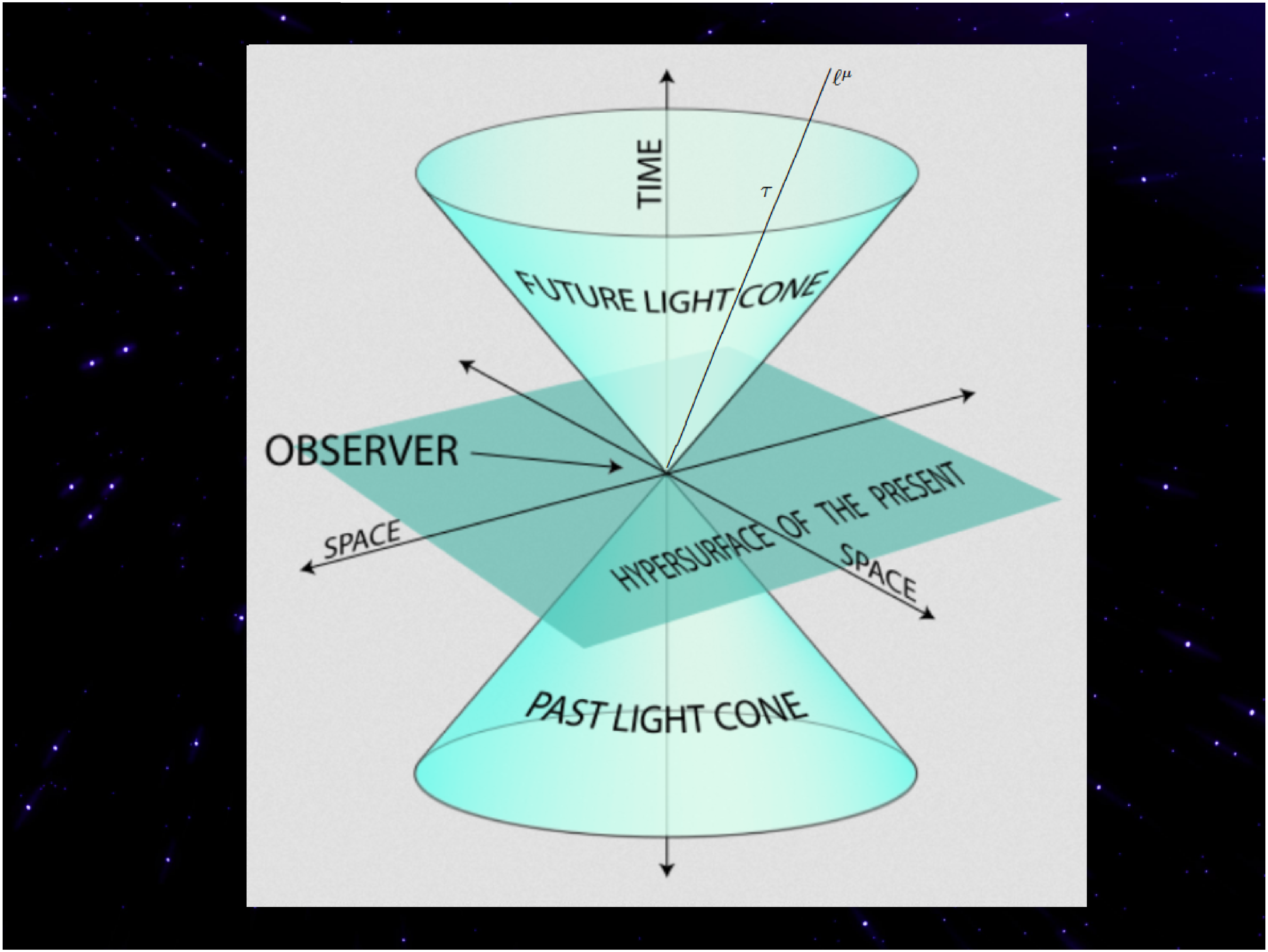
4D Tensor Supermultiplet

$$(\varphi,\ B_{\mu\nu},\ \chi_a)$$

$$\mathrm{D}_a \varphi = \chi_a \quad ,$$

$$\mathrm{D}_a B_{\mu\nu} = -\tfrac{1}{4}([\gamma_\mu , \gamma_\nu])_a{}^b \chi_b \quad ,$$

$$\mathrm{D}_a \chi_b = i\,(\gamma^\mu)_{a\,b}\,\partial_\mu \varphi - (\gamma^5 \gamma^\mu)_{a\,b}\,\epsilon_\mu{}^{\rho\,\sigma\,\tau}\partial_\rho B_{\sigma\,\tau} \quad .$$



ℓ^μ
τ
TIME
FUTURE LIGHT CONE
OBSERVER
SPACE
HYPERSURFACE OF THE PRESENT
SPACE
PAST LIGHT CONE

Valise Formulation

4D Chiral Supermultiplet

$$(\boldsymbol{A},\ \boldsymbol{B},\ \boldsymbol{\psi_a},\ \boldsymbol{F},\ \boldsymbol{G})$$

$$\mathrm{D}_a A = \psi_a \quad , \quad \mathrm{D}_a B = i\,(\gamma^5)_a{}^b \psi_b \quad ,$$

$$\mathrm{D}_a F = (\gamma^0)_a{}^b\,\psi_b \quad , \quad \mathrm{D}_a G = i\,(\gamma^5\gamma^0)_a{}^b\,\psi_b \quad ,$$

$$\mathrm{D}_a \psi_b = i\,(\gamma^0)_{ab}\,(\,\partial_\tau A\,) - (\gamma^5\gamma^0)_{ab}\,(\,\partial_\tau B\,) - i C_{ab}\,(\,\partial_\tau F\,) + (\gamma^5)_{ab}\,(\,\partial_\tau G\,) \quad ,$$

4D Vector Supermultiplet

$(\boldsymbol{A_\mu}, \boldsymbol{\lambda_a}, \mathbf{d})$

$$\mathrm{D}_a A_i = (\gamma_i)_a{}^b \lambda_b \quad , \quad \mathrm{D}_a \mathrm{d} = i(\gamma^5 \gamma^0)_a{}^b \lambda_b \quad ,$$

$$\mathrm{D}_a \lambda_b = -i\,(\gamma^0 \gamma^i)_{ab}\,(\partial_\tau A_i) + (\gamma^5)_{ab}\,(\partial_\tau \mathrm{d}) \quad .$$

Valise Formulation

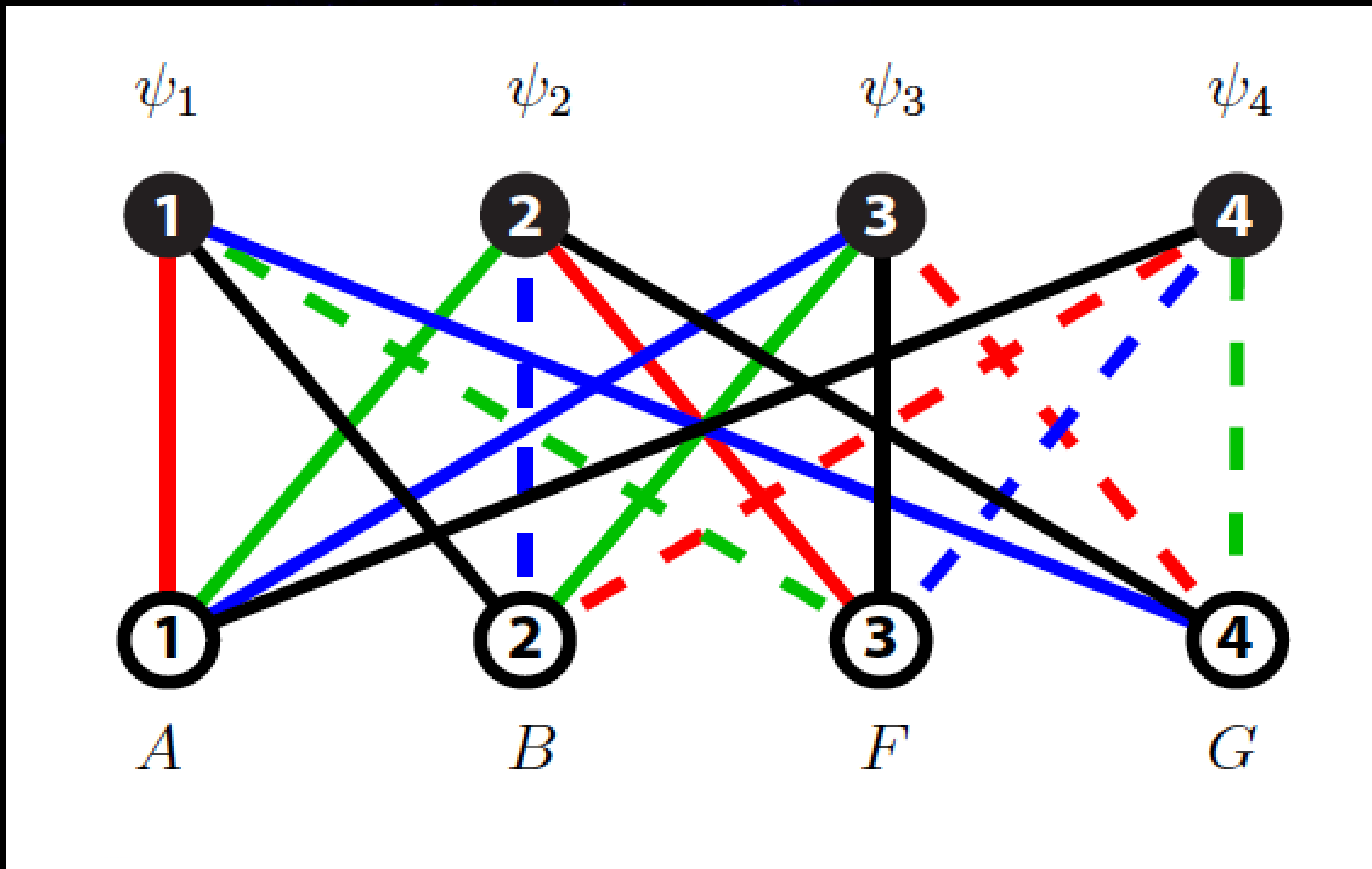
4D Tensor Supermultiplet

$$(\varphi,\, B_{\mu\,\nu},\, \chi_a)$$

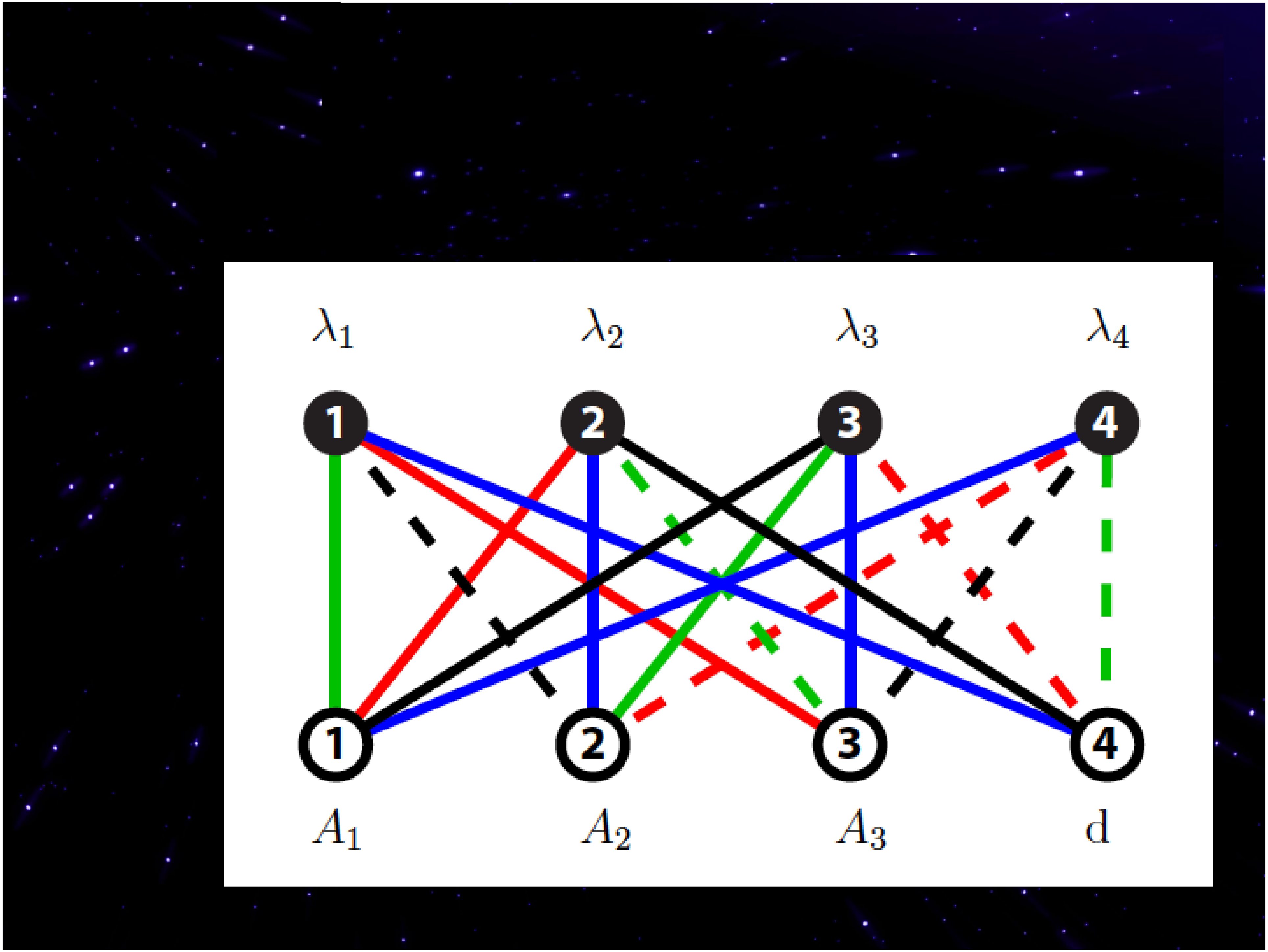
$$\mathrm{D}_a \varphi = \chi_a \quad , \quad \mathrm{D}_a B_{i\,j} = -\frac{1}{4}([\gamma_i,\, \gamma_j])_a{}^b \chi_b \quad ,$$

$$\mathrm{D}_a \chi_b = i(\gamma^0)_{ab}\, \partial_\tau \varphi - (\gamma^5 \gamma^i)_{ab}\, \epsilon_i{}^{j\,k} \partial_\tau B_{j\,k} \quad .$$

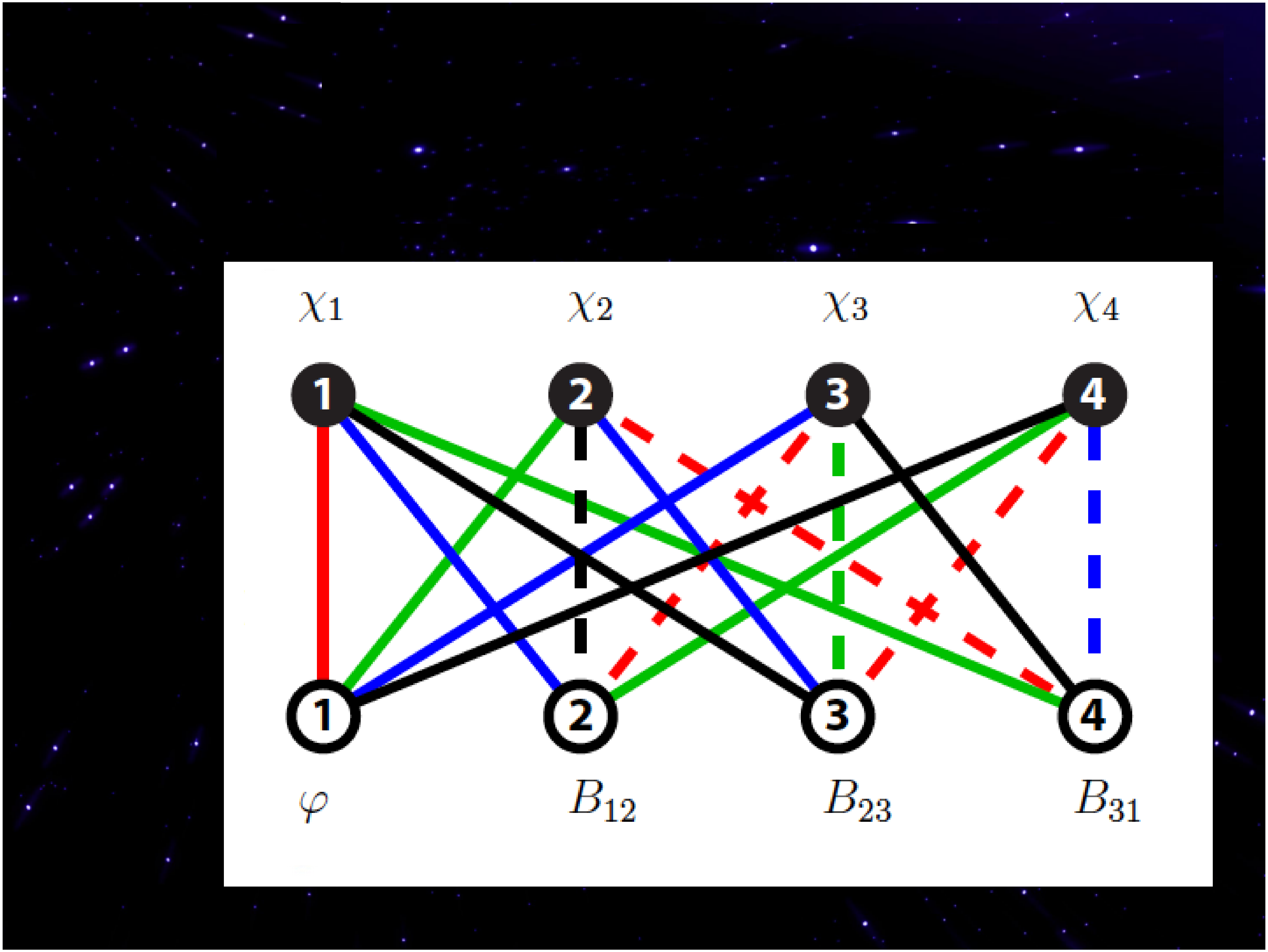
Valise Formulation



ψ_1
ψ_2
ψ_3
ψ_4
1
2
3
4
1
2
3
4
A
B
F
G



λ_1
λ_2
λ_3
λ_4
1
2
3
4
1
2
3
4
A_1
A_2
A_3
d



χ_1
χ_2
χ_3
χ_4
1
2
3
4
1
2
3
4
φ
B_{12}
B_{23}
B_{31}

The ‘Garden Algebra’

$$\mathrm{D_I}\Phi_i \;=\; i\,(\mathrm{L_I})_{i\hat{k}}\Psi_{\hat{k}}$$

$$\mathrm{D_I}\Psi_{\hat{k}} \;=\; (\mathrm{R_I})_{\hat{k}i}\,(\partial_\tau\Phi_i)$$

$$(\mathrm{L_I})_i{}^{\hat{j}}\,(\mathrm{R_J})_{\hat{j}}{}^{k} + (\mathrm{L_J})_i{}^{\hat{j}}\,(\mathrm{R_I})_{\hat{j}}{}^{k} = 2\,\delta_{\mathrm{IJ}}\,\delta_i{}^{k} \;,$$

$$(\mathrm{R_J})_{\hat{i}}{}^{j}\,(\mathrm{L_I})_j{}^{\hat{k}} + (\mathrm{R_I})_{\hat{i}}{}^{j}\,(\mathrm{L_J})_j{}^{\hat{k}} = 2\,\delta_{\mathrm{IJ}}\,\delta_{\hat{i}}{}^{\hat{k}} \;,$$

$$(\mathrm{R_I})_{\hat{j}}{}^{k}\,\delta_{ik} = (\mathrm{L_I})_i{}^{\hat{k}}\,\delta_{\hat{j}\hat{k}} \;.$$

The 'Garden Algebra'

$$\gamma^{I} = \begin{bmatrix} 0 & L^{I} \\ R^{I} & 0 \end{bmatrix}$$

$$\gamma^{I}\gamma^{J} + \gamma^{J}\gamma^{I} = 2\,\delta^{IJ}\,\mathbf{I}$$

For a fixed value of $\mathcal{N}$ there is a minimum value $d_{\mathcal{N}}$ such that $d_{\mathcal{N}} \times d_{\mathcal{N}}$ matrices faithfully represent this algebra. With $\mathcal{N} = 8\,m + n$, $1 \leq n \leq 8$ and using the definition if $\mathcal{N} = 8\,k \rightarrow m = k - 1$ for $k = 1, 2, 3, \ldots \infty$, this minimum value is shown in the following table

$$\mathrm{d}_{\mathcal{N}} = 16^{m}\, F_{\mathcal{R}\mathcal{H}}(n)$$

n	$F_{\mathcal{R}\mathcal{H}}(n)$
1	1
2	2
3	4
4	4
5	8
6	8
7	8
8	8

L-Matrix Decomposition

$$(\mathrm{L_I})_i{}^{\hat{k}} = (\mathcal{S}^{(\mathrm{I})})_i{}^{\hat{\ell}} (\mathcal{P}_{(\mathrm{I})})_{\hat{\ell}}{}^{\hat{k}} \quad , \qquad \text{for each fixed I} = 1, 2, 3, 4.$$

$$(\mathcal{S}^{(\mathrm{I})})_i{}^{\hat{\ell}} = \begin{bmatrix} (-1)^{b_1} & 0 & 0 & 0 \\ 0 & (-1)^{b_2} & 0 & 0 \\ 0 & 0 & (-1)^{b_3} & 0 \\ 0 & 0 & 0 & (-1)^{b_4} \end{bmatrix} \quad \leftrightarrow \quad \left(\mathcal{R}^{\mathrm{I}}\right)_b = \sum_{i=1}^{4} b_i \, 2^{i-1}$$

Taking A Second Look

1 Chiral Multiplet Matrices

$L_1 = (10)_b(2\,4\,3)$, $L_2 = (6)_b(1\,3\,4)$, $L_3 = (0)_b(1\,4\,2)$, $L_4 = (12)_b(1\,2\,3)$

$R_1 = (12)_b(2\,3\,4)$, $R_2 = (10)_b(1\,4\,3)$, $R_3 = (0)_b(1\,2\,4)$, $R_4 = (9)_b(1\,3\,2)$

2 Vector Multiplet Matrices

$L_1 = (10)_b(1\,2\,4\,3)$, $L_2 = (12)_b(2\,3)$, $L_3 = (0)_b(1\,4)$, $L_4 = (6)_b(1\,3\,4\,2)$

$R_1 = (12)_b(1\,3\,4\,2)$, $R_2 = (10)_b(2\,3)$, $R_3 = (0)_b(1\,4)$, $R_4 = (13)_b(1\,2\,4\,3)$

3 Tensor Multiplet Matrices

$L_1 = (14)_b(2\,3\,4)$, $L_2 = (2)_b(1\,4\,3)$, $L_3 = (4)_b(1\,2\,4)$, $L_4 = (8)_b(1\,3\,2)$

$R_1 = (14)_b(2\,4\,3)$, $R_2 = (2)_b(1\,3\,4)$, $R_3 = (4)_b(1\,4\,2)$, $R_4 = (8)_b(1\,2\,3)$