Branched covers of $\mathbb{P}^1$ and $\mathbb{C}$

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1 Abstract

Prerequisites: Undergraduate level topology, graduate courses in complex analysis and group theory.

This course is designed to think about branched coverings of $\mathbb{P}^1$ and $\mathbb{C}$ from various perspectives. We will start with the basics: state the uniformization theorem, introduce the Riemann-Hurwitz formula and braid representations. We will then spend time on specific families of branched covers, the first three of which are algebraic curves.

1. **Hyperelliptic curves**: definition, known results, examples
2. **Dynatomic curves** over $\mathbb{C}$ and $\mathbb{F}_p$: definition and properties, differences between the characteristic 0 and characteristic p cases
3. $\mathbf{Per_n}$- the family of degree 2 rational maps with a critical point of period n: definition and open problems. This is an important curve in complex dynamics, and is related to Gleason polynomials.
4. Transcendental entire functions. Here we will state the **type problem** and examine when it arises. We will introduce the Speiser graph of an entire function and discuss Nevanlinna's theory of how it can be used to construct prescribed polynomial approximations.

If time permits, we will also introduce topological branched coverings and discuss the theory of Bill Thurston that describes when a topological branched covering is "equivalent" in a suitable sense to a holomorphic branched covering.