Worst Election Ever in Russia?

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December 18, 2016

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	Abstract
Analysis shows extensive signs of exter	nsive frauds in the Russian 2016 Duma election. The
frequency and magnitude frauds are we	orse than in any Russian national election since 2000.

Because there was a Stalinist era it is an exaggeration to say that the Russian *Duma* election in 2016 was the worst election ever in Russia, but it is no exaggeration to say the 2016 election had the most extensive and largest magnitudes of frauds since the 2000 presidential election. Many diagnostics can support that claim, but here we show only a few results in support using methods in the Election Forensics Toolkit (Hicken and Mebane 2015; Mebane 2015), including the finite mixture likelihood model (Mebane 2016). We also use results from a method developed by Rozenas (forthcoming).

In the September 2016 legislative (Duma) election Russians cast two ballots, one for a party competing for seats determined nationally by proportional representation (PR) and one for a candidate running for a single-member district (SMD) seat. A similar mixed voting system had previously been used between 1993 and 2003. The State Duma consists of 450 deputies elected for five-year terms. In 2016 half of the deputies were elected by PR with 5% threshold and another half in plurality SMDs. The 2011 Duma elections were marred by allegations of fraud (Enikolopov, Korovkin, Petrova, Sonin and Zakharov 2013) and massive protests, but in 2016 authorities referred to the importance of genuine popular support and clean elections. Since 2011 the electoral system was modified, the parliamentary threshold was reduced from 7% to 5%, and independent candidates were allowed to participate. Appointment of a new head of the Ella Pamfilova having a reputation as a human rights advocate was aimed to build greater trust and credibility in the upcoming elections (Roth 2016).

The legal framework remains complex restricting candidate registration, formation of party blocks, campaigning, and electoral observation (OSCE 2016). New laws such as anti-protest law, imposing heavy fines on organizers or participants of unsanctioned demonstrations and anti-NGO laws, serve as additional checks on the civil society (Gregory 2016).

¹Such laws include a "foreign agents" law—any NGO receiving foreign funding and engaged in "political activity" is required to register as foreign agent— and the undesirable organizations law targeting foreign organizations that are considered threatening to Russia's national security.

In 2016 the Kremlin enacted a "voter demobilization" campaign aimed to discourage critically-minded voters from participation by shifting elections to the fall when many Russians are on vacation, harvesting crops or students are back to school. Many discouraged voters chose to ignore or to boycott the election (Rezunkov 2016). The Kremlin used administrative resources—the regional political machines—to mobilize its own supporters and to provide overwhelming victory to the party of power.

Putin's high approval ratings suggests that no special measures would have been needed, but over the electoral campaign preelection polls reported a sharp decline in the United Russia party's ratings: from 40% in early 2016 to 31% by the end of August according to Russia's leading independent polling agency Levada Center (Sharkov 2016). Unfavorable election polls for the party of power provoked a sharp reaction from Kremlin. Two weeks before the election the justice ministry labeled Levada Center a "foreign agent", making its future uncertain and sending a strong signal to other Russian pollsters (Vladimirov 2016).

New legal limitations on domestic observers reduced election observation. For instance at most two observers were allowed per party/candidate, all observers were assigned to a specific precinct, and observers under the guise of media representatives (one of the solutions used in the past to involve non-partisan observers—non-partisan citizen observation doesn't exist in Russia) were no longer allowed (Law 2016). Often traditional observers were substituted by "phony" observers from organizations aligned with the state apparatus.

The official turnout for 2016 elections of 48% was the lowest in the Russian recent history (62% in 2011 and 59% in 2007). Out of 14 parties listed in the ballot only four traditional *Duma* parties managed to surpass the 5% threshold: United Russia with 54.2%, Communists (13.34%), LDPR (13.14%) and Just Russia (6.22%). In SMDs with 4437 candidates participating, the major winner was United Russia, which gained the majority of seats winning in 203 out of 206 districts it was running in, followed by the Communists and Just Russia earning 7 seats each, LDPR (5 seats) and the remaining seats divided

between smaller parties and independents. In all 343 seats out of 450 seats, i.e. 76% of the seats went to United Russia.

Were these elections clean? Observers report that unlike the previous 2011 election, there were fewer cases related to vote rigging or conflicts between the observers and election committees. While OSCE assessed the voting process as good in 97 percent of observations, the counting process was worse: it was assessed by the observers as bad in 23 percent of the polling stations observed (OSCE 2016). In selected precincts the abuse with administrative resources and illegal campaigning, ballot stuffing and carousel voting, violation of the rights of observers, commission members and representatives of the media, violation counting procedures, ghost voters in the registration lists, protocol tampering were reported (Golos 2016). According to Golos ballot stuffing was reported in Moscow, St. Petersburg, and many other regions including republics such as Chechnya, Bashkortostan, Dagestan, Tatarstan. Those excessive figures for United Russia and turnout may reflect both the extent of popular support for Kremlin policy, but more likely the strength of the governors' political machines to provide a favorable to Kremlin electoral outcome. As always, Chechnya with its overwhelming level of support for United Russia 96% and turnout 94% is a leading outlier (Fuller 2016).

Indirect evidence suggests pronounced election fraud. According toexit polls by VCIOM (Russia's leading polling agency closely aligned with the state), United Russia should have received 44.5%—a ten percent discrepancy from the officially reported vote share (Russian Public Opinion Research Center 2016).

Similar to the previous Russian elections, in 2016 the regional governors were responsible for mobilizing their regional "political machines" to provide a favorable electoral result and to signal their loyalty status to Kremlin (Kalinin 2016a). Governors were interested in boosting the level of turnout to provide their respective regions with more mandates in the lower chamber and they were incentivized to boost the United Russia's support to meet the Kremlin's demand (Mebane and Kalinin 2010). Consequently

we expect ballot stuffing and protocol tampering. Election anomalies in turnout and in United Russia's vote shares serve as a basic "signaling" mechanism by which the governor reports loyalty to the Center (Kalinin 2016b; Kalinin and Mebane 2011).

Such signaling is apparent in Table 1, which for elections from 2000 through 2016 reports mean statistics for variables that indicate whether the last digit of the rounded percentage of a count is zero or five (Hicken and Mebane 2015). Table 1 shows statistics for the federal proportional representation (PR) vote in *Duma* elections and for the federal vote in presidential elections. In every year the statistics for turnout differ significantly from the values expected in the absence of frauds (such results are shown in red). The statistics for United Russia vote proportions differ significantly only from 2004 on. The simplest explanation why signaling expanded to vote proportions in 2004 is that only then did the incentives to benefit Putin and United Russia with, potentially, nationwide scope specifically begin. Before then only the governors' interests in boosting turnout to increase their regions' mandates are apparent.

*** Table 1 about here ***

Figure 1 compares district-specific results between 2003 and 2016. The statistics are significantly elevated for turnout in both elections, but in 2016 many more statistics are elevated for the district winner's vote proportions.

*** Figure 1 about here ***

Using a resampled kernel density method (Rozenas forthcoming) supports both the general signaling interpretation (see also Rundlett and Svolik 2016) and the specific finding of an increase in signaling behavior since 2004. As Figures 2–4 show, the percentage of precincts with "fraudulent" election results increases from .06 percent in 2000 up to 1.12 percent in 2008, falling to .85 percent in 2016. In 2000 and in 2003 vote proportions occur suspiciously often for relatively few vote proportion values (and few "signaling" values)

while in 2004 and in subsequent years there are many more distinct suspicious values. In 2016 too many of the vote shares received by United Russia in individual precincts are greater than 50 percent and evenly divisible by five: too many are 55%, 65%, 70%, 75%, 80%, 85%, 90%, 95% or 100%.

*** Figures 2, 3 and 4 about here ***

Estimates of the finite mixture "frauds" model (Mebane 2016; Klimek, Yegorov, Hanel and Thurner 2012), using polling station observations, support the idea that the 2016 election exhibits the most fraud since 2000, but they also suggest the manner in which frauds are committed is stable over time. Point estimates \hat{f}_i , \hat{f}_e , $\hat{\alpha}$, $\hat{\theta}$, $\hat{\tau}$ and $\hat{\nu}$ are reported in Table 2 and Figures 5 and 6 (Mebane 2016). Table 2 shows that the probabilities of frauds, \hat{f}_i and \hat{f}_e , are greater in the 2016 PR vote than in any previous national election going back to 2000. In fact, $\hat{f}_i = .22$ in 2016 PR is about twice as large as the next largest \hat{f}_i value and $\hat{f}_e = .022$ in 2016 PR is about seven times as large as the next largest \hat{f}_e value. Figure 5(a) shows that frauds estimates for the 2016 SMD votes are frequently large and sometimes larger than the values estimated for the 2016 PR vote.

*** Table 2 and Figures 5 and 6 about here ***

Despite the volatility in the probabilities of frauds, the mechanism used to effect the fraudulent votes appears to be stable over time. Parameter α indicates whether vote manufacturing or vote stealing predominate. If $\alpha = 1$ then both processes are equally

²Data source information: Central Election Commission of the Russian Federation (2013).

³In every election shown in Table 2 the party in focus in the analysis is the party that received the most votes nationally. For Figure 5 the candidate in focus in each district is the candidate with the most votes in the district.

 $^{^4}f_{\rm i}$ and $f_{\rm e}$ are respectively the probabilities that each precinct is affected by incremental or extreme frauds. Smaller values of α mean that larger fractions of votes are shifted from opposition to the leading candidate. A higher value of θ implies that incremental fraud garners a higher number of votes for the leading party. τ and ν are respectively the mean turnout proportion and mean proportion of votes for the leading party in the absence of fraud. Table 2 also includes reports of the likelihood ratio test statistic for the hypothesis that there are no frauds (i.e., that $f_{\rm i}=f_{\rm e}=0$), along with the number of polling station observations for each election.

affecting votes. If $\alpha < 1$ then vote stealing is more important, and if $\alpha > 1$ manufacturing votes from nonvoters is more important (Mebane 2016, 8–9). $\hat{\alpha}$ suggests that vote manufacturing is the dominant fraud mechanism in every election since 2000.⁵ Indeed, $\hat{\alpha}$ has the same values in many of the elections: $\hat{\alpha} = 1.7$ in 2004, 2007, 2008 and 2016 ($\hat{\alpha} = 1.8$ in 2011); $\hat{\alpha} = 3.3$ in 2000 and 2003 ($\hat{\alpha} = 3.4$ in 2012). In terms of α , at least, vote manufacturing appears to operate at two different scales across elections. Despite the limited variation in $\hat{\alpha}$, $\hat{\theta}$ varies more than $\hat{\alpha}$ does, so the precise number of fraudulent votes manufactured varies considerably across elections.

More votes are produced by frauds in 2016 than in previous Russian elections. Estimates of the number of votes produced by incremental and extreme frauds (Mebane 2016, 13) show nearly two million fraudulent votes in the 2016 PR vote, amounting to 3.6% of the recorded votes (Table 3). Both the number and percentage of fraudulent votes are greater than in other elections going back to 2000. Figures 5(d,e) show that in the 2016 SMD votes the numbers and proportions of fraudulent votes are also frequently large. Comparisons with Figures 6(d,e) show that many fewer votes were produced in particular by extreme frauds in 2003 than in 2016.

*** Table 3 about here ***

To convey information about the geographic dispersion of frauds in the 2016 PR votes, Figure 7 shows hotspots and Figure 8 shows local spatial clustering patterns of conditional frauds probabilities \hat{f}_{ii} and \hat{f}_{ei} (Mebane 2016, 12–13). See Hicken and Mebane (2015, 13-14) and Mebane (2015, 8-12) for hotspot/clustering methods explanations and for color legends. Briefly, red dots are locations where geographic concentrations of relatively high frauds probabilities occur.

*** Figures 7 and 8 about here ***

⁵In Figure 5(b) usually $\hat{\alpha} > 1$ for the 2016 SMD votes. In Figure 6(b) $\hat{\alpha} > 1$ in just more than half of the districts for the 2003 SMD votes.

⁶Hotspots and clusters are estimated using estimates of the probability that that each precinct i = 1, ..., n is a case of no fraud (\hat{f}_{0i}) , incremental fraud (\hat{f}_{ii}) or extreme fraud (\hat{f}_{ei}) (Mebane 2016, 12).

We apply the Getis-Ord G_i analysis of hotspots (Ord and Getis 1995) to measure whether the mean of \hat{f}_{ii} and \hat{f}_{ei} values geographically close to observation i differs from the global mean. It seems that in both cases of extreme and incremental fraud high values (red dots) are almost uniformly scattered across the Russian territory suggesting that anomalies are clustered in many small localities across Russia (in the case of incremental frauds two big clusters of anomalies draw our attention, these clusters are located in Altaiskii krai, Republic Marii El and Amurskaya oblast').

Figure 9 displays regional averages of frauds probabilities: brighter red color indicates higher levels of election fraud. The figure suggests election fraud is much more frequent in the south of Russia, especially, the republics of the North Caucasus, Russian regions located on the south from Moscow, and Volga region republics such as Tatarstan and Mordoviya. In addition to this several Siberian regions such as Yamalo-Nenets autonomous district, Tyumenskaya Oblast', Kemerovskaya Oblast' and Tyva Republic expose elevated levels of election fraud. Our findings with respect to geographical variation of fraud are supported by data analysis performed by other scholars (see e.g. Kireev 2016).

*** Figure 9 about here ***

Our findings suggest that from a comparative perspective the quality of the most recent *Duma* elections seems be the worst out of all elections of the 2000s. The estimated magnitude of election fraud and its spread across the country suggests that Russian election anomalies are deeply embedded in both the regional and local levels. As the Russian regime becomes more authoritarian, the quality of elections also suffers.

References

- Benjamini, Yoav and Yosef Hochberg. 1995. "Controlling the False Discovery Rate: A Practical and Powerful Approach to Multiple Testing." *Journal of the Royal Statistical Society, Series B* 57(1):289–300.
- Central Election Commission of the Russian Federation. 2013. "Elections and referendums." URL http://www.vybory.izbirkom.ru/.
- Enikolopov, Ruben, Vasily Korovkin, Maria Petrova, Konstantin Sonin and Alexei Zakharov. 2013. "Field experiment estimate of electoral fraud in Russian parliamentary elections." Proceedings of the National Academy of Sciences 110(2):448–452.
- Fuller, Liz. 2016. "Evidence Of Blatant Violations Calls Into Question Validity Of Elections In North Caucasus.". URL http://www.rferl.org/a/caucasus-report-duma-elections-blatant-violations/28007463.html.
- Golos. 2016. Preliminary Statement on Election Observation on September, 18 2016. Technical report Golos. URL http://www.golosinfo.org/ru/articles/117564.
- Gregory, Paul Roderick. 2016. "Putin Changes September Election Rules To Prop Up His 'United Russia' Party.".
- Hicken, Allen and Walter R. Mebane, Jr. 2015. "A Guide to Election Forensics." Working paper for IIE/USAID subaward #DFG-10-APS-UM, "Development of an Election Forensics Toolkit: Using Subnational Data to Detect Anomalies".
- Kalinin, Kirill. 2016a. "Signaling Games of Election Fraud." URL: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2836775, Working paper.
- Kalinin, Kirill. 2016b. "Validating Precinct-Level Measures of Fraud: Evidence from the Russian Electoral Cycle 2011-2012." Paper presented at the 2016 Annual Meeting of the Midwest Political Science Association, Chicago, April 7–10, 2016.
- Kalinin, Kirill and Walter R. Mebane, Jr. 2011. "Understanding Electoral Frauds through Evolution of Russian Federalism: from "Bargaining Loyalty" to "Signaling Loyalty"." Paper presented at the 2011 Annual Meeting of the Midwest Political Science Association,

- Chicago, IL, March 31–April 2.
- Kireev, Alexander. 2016. "My Estimate of Falsification Levels in the Duma Elections." blog
 . URL http://kireev.livejournal.com/?skip=20.
- Klimek, Peter, Yuri Yegorov, Rudolf Hanel and Stefan Thurner. 2012. "Statistical Detection of Systematic Election Irregularities." *Proceedings of the National Academy of Sciences* 109(41):16469–16473.
- Law. 2016. Federal Law (15 Feb. 2016). Technical Report N29-FZ Russian Federation. URL https://rg.ru/2016/02/17/vybori-dok.html.
- Mebane, Jr., Walter R. 2015. "Election Forensics Toolkit DRG Center Working Paper." Working paper for IIE/USAID subaward #DFG-10-APS-UM, "Development of an Election Forensics Toolkit: Using Subnational Data to Detect Anomalies".
- Mebane, Jr., Walter R. 2016. "Election Forensics: Frauds Tests and Observation-level Frauds Probabilities." Paper presented at the 2016 Annual Meeting of the Midwest Political Science Association, Chicago, April 7–10, 2016.
- Mebane, Jr., Walter R. and Kirill Kalinin. 2010. "Electoral Fraud in Russia: Vote Counts Analysis using Second-digit Mean Tests." Paper prepared for the 2010 Annual Meeting of the Midwest Political Science Association, Chicago, IL, April 22–25.
- Ord, J. K. and Arthur Getis. 1995. "Local Spatial Autocorrelation Statistics: Distributional Issues and an Application." *Geographical Analysis* 27(4):286–306.
- OSCE. 2016. Russian Federation State Duma Elections, 18 September 2016. Statement of Preliminary Findings and Conclusions. Technical report OSCE.
- Rezunkov, Victor. 2016. "The Most Boring Campaign (Interview with Grigorii Golosov).".

 URL http://www.svoboda.org/a/27991655.html.
- Roth, Andrew. 2016. "Meet the Woman Who Says She's Going to Fix Russia's Rigged Elections." The Washington Post. URL https://www.washingtonpost.com/world/europe/meet-the-woman-who-says-shes-going-to-fix-russias-rigged-elections/2016/05/14/13f9ed7e-0e36-11e6-bc53-db634ca94a2a_story.html.

- Rozenas, Arturas. forthcoming. "Detecting Election Fraud from Irregularities in Vote-Share Distributions." to appear in *Political Analysis*.
- Rundlett, Ashlea and Milan W. Svolik. 2016. "Deliver the Vote! Micromotives and Macrobehavior in Electoral Fraud." *American Political Science Review* 110(1):180–197.
- Russian Public Opinion Research Center. 2016. "Russian State Duma Elections: VCIOM Exit Poll Data.". URL http://wciom.com/index.php?id=61&uid=1305.
- "Putin's Polls Sharkov, Damien. 2016. United Russia Drops In Ahead URL http://www.newsweek.com/ of September Elections.". putins-united-russia-drops-polls-ahead-september-elections-494887.
- Vladimirov, Victor. 2016. Russian Pollster: Authorities Want to Destroy NGOs. Technical report.

Table 1: "Signaling" Digit Tests for National Votes

	2000	2003 PR	2004	2007
Turnout	0.221	0.217	0.236	0.228
	(0.218, 0.223)	(0.214, 0.22)	(0.233, 0.239)	(0.225, 0.23)
United Russia	0.202	0.202	0.207	0.21
	(0.199, 0.204)	(0.199, 0.204)	(0.204, 0.209)	(0.207, 0.212)
	2008	2011	2012	2016 PR
Turnout	$2008 \\ 0.232$	2011 0.219	2012 0.22	2016 PR 0.225
Turnout				
Turnout United Russia	0.232	0.219	0.22	0.225

Note: the statistic is the mean of a variable indicating whether the last digit of the rounded percentage of votes for the referent party or candidate at each polling station is zero or five. Values in parentheses are nonparametric bootstrap confidence intervals.

Table 2: Finite Mixture Model Parameter Estimates for Russian Elections

Election	$\hat{f_{ m i}}$	$\hat{f}_{ m e}$	\hat{lpha}	$\hat{ heta}$	$\hat{ au}$	$\hat{ u}$	LR	n
2000 President	.033	.000032	3.3	.71	.71	.54	22,286	91,306
2003 Duma PR	.16	.0033	3.3	.27	.58	.36	106,850	95,077
2004 President	.049	.000087	1.7	.44	.69	.72	20290	95,424
2007 Duma	.040	.00016	1.7	.53	.67	.66	18694	95,802
2008 President	.013	.0000017	1.7	.53	.76	.70	586	96,242
2011 Duma	.12	.0032	1.8	.36	.61	.48	69244	95,166
2012 President	.084	.0020	3.4	.35	.65	.65	55352	95,413
2016 Duma PR	.22	.022	1.7	.27	.48	.49	233724	94,987

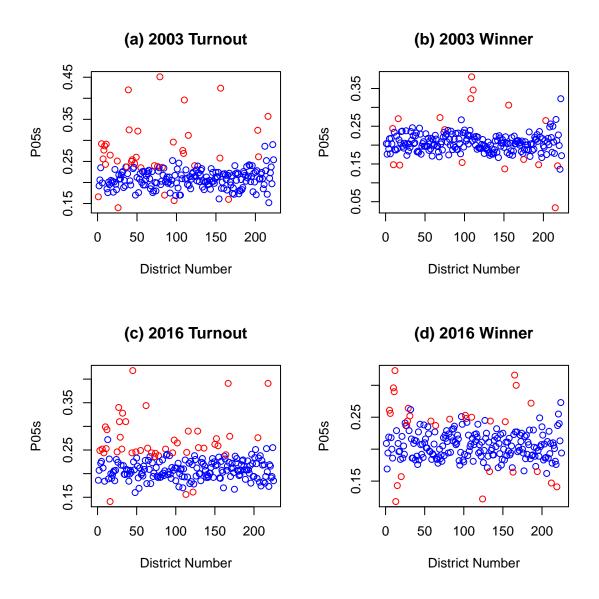
Note: LR is the likelihood ratio test statistic for the hypothesis that there are no frauds (i.e., that $f_i = f_e = 0$). n is the number of polling station observations.

Table 3: Estimated Fraudulent Vote Counts and Proportions for Russian Elections

Election	$M_{ m i}$	$M_{ m e}$	$p_{ m i}$	$p_{ m e}$	$p_{\rm i} + p_{\rm e}$
2000 President	135,061	1,452	.00187	.0000202	.00190
2003 Duma PR	256,759	$185,\!278$.00430	.00311	.00741
2004 President	203,955	4,951	.00297	.0000721	.00304
2007 Duma	270,490	11,914	.00395	.000174	.00413
2008 President	84,933	113	.00116	.00000155	.00116
2011 Duma	680,082	$260,\!254$.0105	.00403	.0146
2012 President	292,339	189,912	.00413	.00268	.00681
2016 Duma PR	739,005	1,080,856	.0145	.0212	.0356

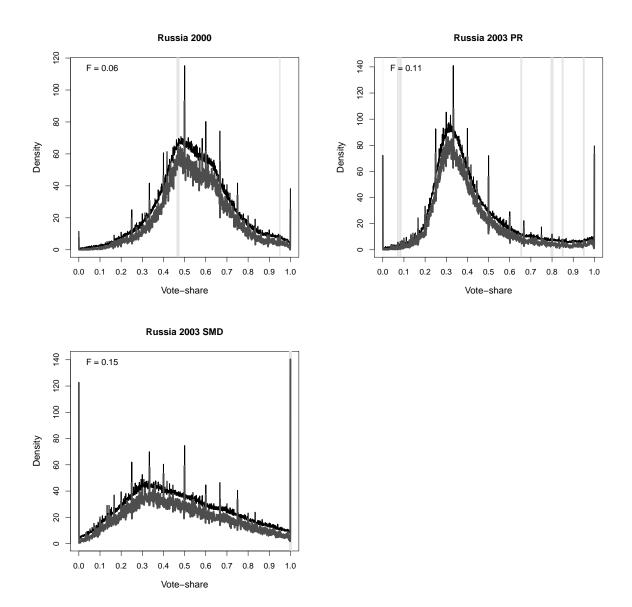
Note: $M_{\rm i},\,M_{\rm e}$ are estimated numbers of votes produced by incremental and extreme frauds; $p_{\rm i},\,p_{\rm e}$ are fraudulent vote counts as proportions of the recorded votes.

Figure 1: "Signaling" Tests by District, Russia 2003 and 2016 SMD Votes



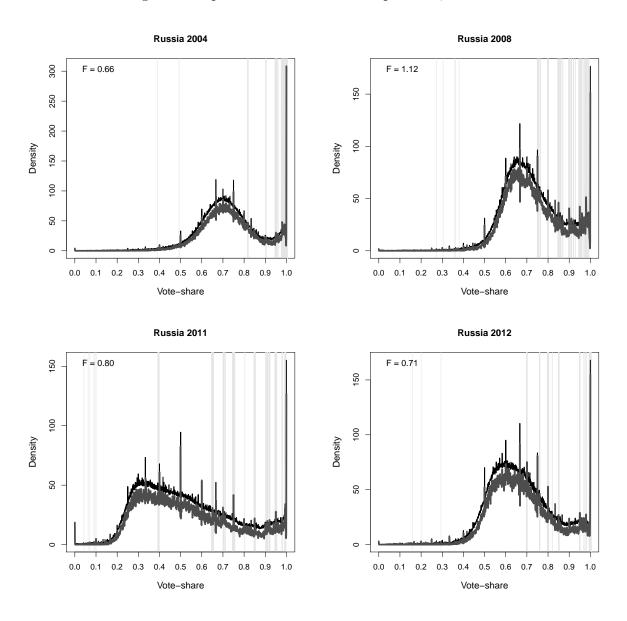
Note: SMD-specific statistics and tests based on polling station observations from 224 (2003) and 225 (2016) districts. "P05s," mean of variable indicating whether the last digit of the rounded percentage of votes for the referent party or candidate is zero or five. Statistics that deviate from the values they are expected to have in the absence of fraud are in red.

Figure 2: "Spikes" Tests for Vote Proportions, 2000–2003



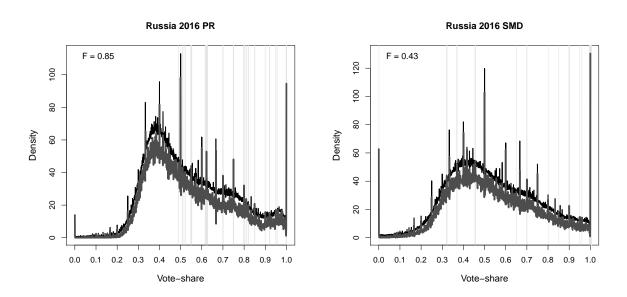
Note: light gray lines show values for which the kernel density of the observed data exceeds the upper envelope of the kernel density of the resampled data (Rozenas forthcoming).

Figure 3: "Spikes" Tests for Vote Proportions, 2004–2012



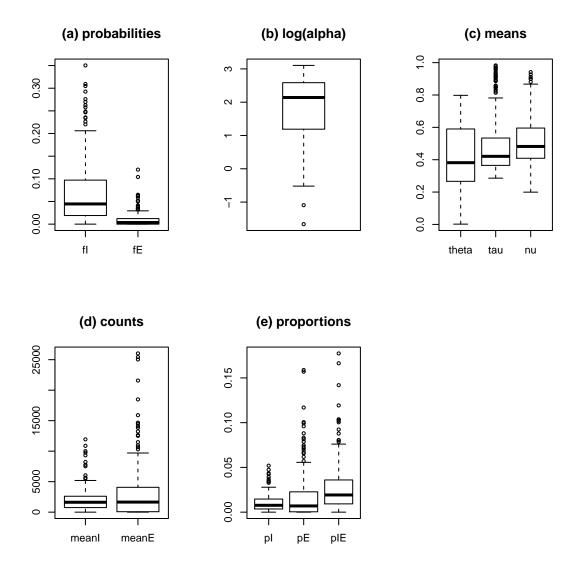
Note: light gray lines show values for which the kernel density of the observed data exceeds the upper envelope of the kernel density of the resampled data (Rozenas forthcoming).

Figure 4: "Spikes" Tests for Vote Proportions, 2016



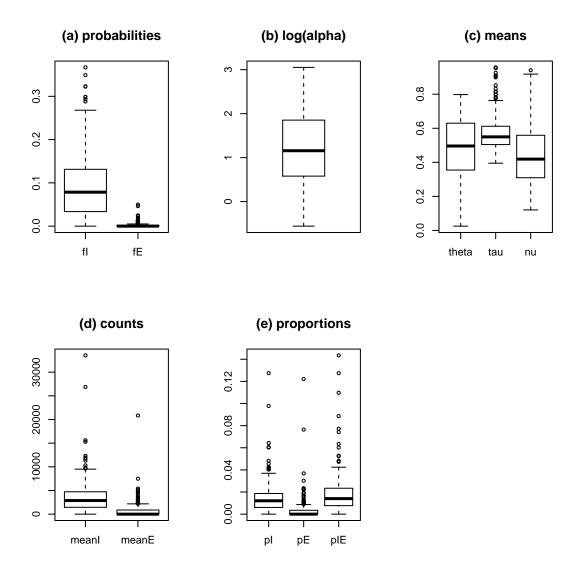
Note: light gray lines show values for which the kernel density of the observed data exceeds the upper envelope of the kernel density of the resampled data (Rozenas forthcoming).

Figure 5: Finite Mixture Model Estimates, Russia 2016 SMD



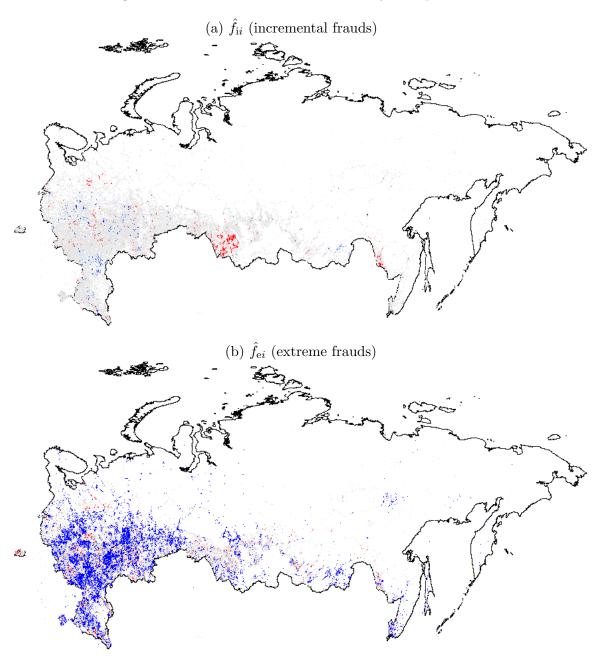
Note: distribution of district-specific estimates over 225 districts; (a) \hat{f}_i , \hat{f}_e ; (b) $\log(\hat{\alpha})$; (c) $\hat{\theta}$, $\hat{\tau}$, $\hat{\nu}$; (d) M_i , M_e ; (e) p_i , p_e , $p_i + p_e$.

Figure 6: Finite Mixture Model Estimates, Russia 2003 SMD



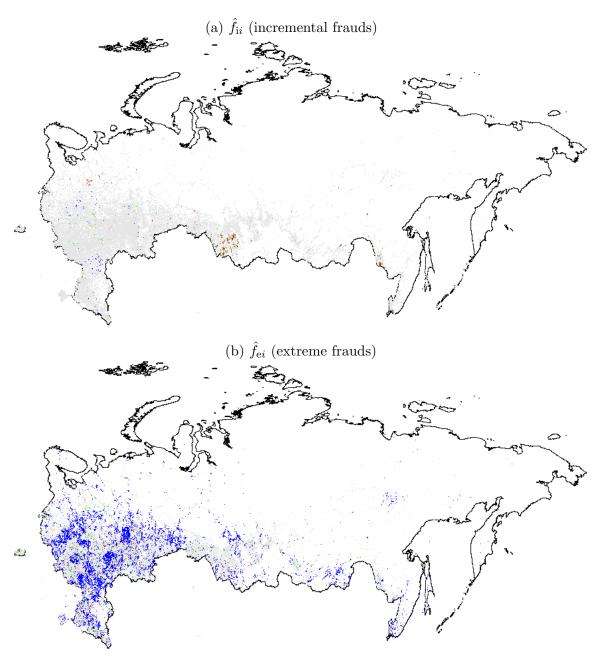
Note: distribution of district-specific estimates over 224 districts; (a) \hat{f}_i , \hat{f}_e ; (b) $\log(\hat{\alpha})$; (c) $\hat{\theta}$, $\hat{\tau}$, $\hat{\nu}$; (d) M_i , M_e ; (e) p_i , p_e , $p_i + p_e$.

Figure 7: Frauds Conditional Probability Hotspots, 2016



Note: polling station fraud probability hotspot analysis using Getis-Ord G_i (Mebane 2015, 8-12). Red colors show areas where local average scores are significantly above the overall average. Blue colors show areas where local average scores are significantly below the overall average. Gray indicates polling station locations that do not differ significantly from average. Significance levels refer to tests adjusted for the false discovery rate (Benjamini and Hochberg 1995).

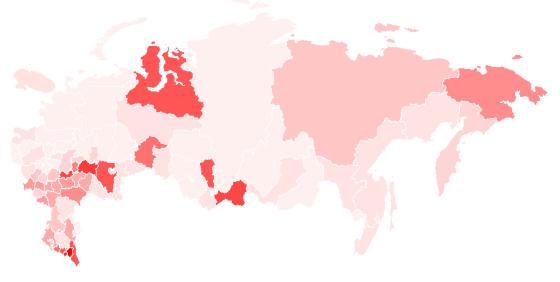
Figure 8: Frauds Conditional Probability Clusters and Outliers, 2016



Note: polling station fraud probability hotspot analysis using local Moran's I_i (Mebane 2015, 8-12). Red, high value among high values; blue, low value among low values; green, low value among high values; orange, high value among low values. Gray indicates polling station locations that do not differ significantly from average. Significance levels refer to tests adjusted for the false discovery rate (Benjamini and Hochberg 1995).

Figure 9: Frauds Conditional Probability Region Averages, 2016

(a) $\hat{f}_{{\rm i}i}$ (incremental frauds region averages)



(b) \hat{f}_{ei} (extreme frauds region averages)

