A Guide to Election Forensics*

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Abstract

Election forensics are an emergent means by which to isolate potential anomalies in and diagnose the accuracy of reported election results, using statistical techniques that can identify patterns and assess their chances of occurring by chance. The purpose of this Guide is to help orient users about how best to use election forensics as one constructive tool among several in election assistance and monitoring efforts, serving as a complement to an Election Forensics Toolkit that implements several of the methods. The Guide discusses the strengths and weaknesses of election forensics relative to other tools for evaluating the integrity of elections, as well as the different types of forensic methods and their respective advantages and limitations. The Guide describes foundations of these methods and the calculations they involve, and it offers guidance on interpreting results, including in a comparative context. The use of the methods is demonstrated through illustrative case studies.

1 Background

Under the best conditions, elections allow citizens to express their preferences, provide information about favored issues, stances and policies, connect voters to their governments, hold representatives and other leaders accountable, and settle social divisions that could otherwise become violent. When elections are widely viewed as free and fair, this outcome reduces the likelihood that winners and losers in the election contests will take actions that undermine democratic foundations with destabilizing effects. If the contestants believe that they can participate in a regular election contest with transparency and fairness, and they endorse the electoral rules, then legitimacy, trust, and stability in society is enhanced and progressively entrenched (Dahl 1971). While elections are not sufficient by themselves to constitute a full, mature democracy (Zakaria 1997), comparative empirical evidence shows that conducting elections—especially if they are accepted and allow true competition accompanied by the prospect of alternation in power—can contribute to a virtuous cycle even under difficult conditions (Lindberg 2006).

Departures from this ideal, however, are not uncommon, especially in new and fragile democracies.

Election malfeasance can take a variety of forms, including the intimidation of voters, violence against candidates and their supporters, and outright election fraud. Regardless of the type of malfeasance, when the electoral process is perceived to be significantly flawed or biased, and the final results suspect or rigged, then the salutary effects of elections can evaporate. Instead of building trust and stability, elections can become corrosive and destabilizing—sapping support for democratic institutions, inflaming suspicion, and stimulating demand for extra-constitutional means of pursuing political agendas. In short, electoral malfeasance can distort and destabilize democracy. Malfeasance can affect electoral outcomes (including which contestants win public office), influence whether or not the results are accepted as legitimate, and shape the extent to which elections produce subsequent political and social stability.

An aim of this guide is to summarize and reflect on ways of mitigating these complications. The focus includes electoral malfeasance surrounding the counting of votes. Such malfeasance ranges from glitches and indications of minor malfeasance in the processes (all too prevalent even in OECD countries), to more serious instances involving actual or apparent manipulation of results, including fraudulent or fictitious figures reported by the government. Instances of malfeasance have been relatively common around the world. During the first half of 2015 alone, for example, allegations of election fraud occurred in Bangladesh, India, Israel, Macedonia, Nigeria, Pakistan, Russia, Togo, the United Kingdom, the United States and Zambia. These allegations run the gamut from minor irregularities in isolated polling locations to systematic fraud at a national scale.

More comprehensively, data compiled by Kelley and Kolev (2010) on national elections conducted in more than 170 countries from 1978-2004 indicate the following:

- 61% of countries experienced some degree of (known) cheating, including fraud related to the tabulation of votes, with 27% of countries exhibiting major problems;
- 25% of the legislative elections and 15% of the presidential elections were plagued by cheating; and
- These issues are persistent, affecting more than half of the elections conducted in 2000 and nearly half in 2002-2004.

An immediate consequence of such malfeasance can be controversy about the results raised by parties and candidates (and their supporters), especially those in the opposition. Data compiled by Lindberg (2006) on national elections held in Africa from 1989–2007 highlights the importance of the perceived integrity of the election process and results:

- Of elections rated as free and fair, 93% were accepted by all of the main losing parties;
- Of elections with minor irregularities, 49% were accepted by all of the main losing parties, 39% exhibited partial acceptance (either by some of the main losing parties, but not all, or not by all of them at first, but eventually), and 12% were not accepted;

- Of elections with substantial irregularities that affected the results, 2% were accepted by all of the main losing parties, 40% exhibited partial acceptance, and 58% were not accepted; and

- Of elections that were rated as not free and fair, none were accepted by all of the main losing parties, 9% exhibited partial acceptance, and 91% were not accepted.

These controversies are hardly limited to public debate and dissent. Instead, they can quickly turn violent. A conspicuous recent example is the large-scale turmoil after Kenya's disputed 2007 presidential election, leading to more than 1,000 deaths. Other similar cases have been observed elsewhere in Africa (e.g., Cote d'Ivoire, Nigeria, Zimbabwe) and around the world.

Given the ubiquity of fraud allegations, what methods are available to observers to assess the credibility of such claims? Can the same methods be used to confirm the fairness of an election in the absence of fraud allegations?

There is an acute need for methods of detecting and investigating fraud. First, accurate information on electoral irregularities can help separate false accusations from true evidence of electoral malfeasance. Second, accurate information about the scope of irregularities can provide election monitors with a better gauge of the quality of elections. Third, accurate information about the geography of malfeasance—the locations where irregularities are observed and how they cluster—can bolster the activities of election monitors and pro-democracy organizations, among other things allowing them to focus their attention and resources more efficiently and helping them to substantiate their assessments of the quality of elections.

Ideally a method for detecting electoral fraud should meet each of the following criteria:

- The method should be sensitive enough to detect anomalies. We want to avoid mislabeling problematic elections as problem-free. In other words, we want to limit false negatives.

- The method should accurately detect anomalies. The method should reveal anomalies when they arise, but produce null results when no anomalies are present. In other words, we want to limit false positives.

- The method should involve systematic observation. In order avoid potential bias, wherever possible we want to analyze electoral results in their entirety, ideally at the most fine-grained level possible. This approach is preferable to limited descriptive evidence and select case studies.

- The method should enable the identification of where, geographically, anomalies have occurred. While it is helpful to know that anomalies exist, ideally methods will identify the locations where those anomalies occur. This facilitates additional analysis and validation and enables more effective responses. Geographic analysis can also take into account the possibility that anomalies may cluster together and be related to other relevant political, cultural or ethnic factors.

- The method should produce estimates of uncertainly, indicating how confident we can be in our conclusions. A method need not yield definitive findings to be worthwhile in this setting. Absent definitive findings, a measure of the extent to which the results are likely to be valid is crucial.

Scholars, aid agencies, and policy makers are devoting increasing attention to detecting election irregularities and trying to improve election processes. Current efforts to promote the integrity of elections around the world are multi-pronged. They include designing electoral systems and procedures in accordance with standards of best practice, promoting the development and capacity of independent election agencies, ballot reform, and on-the-ground monitoring of elections by domestic and foreign observers.

The last of these—on-the-ground monitoring—addresses a direct form of malfeasance: manipulating the results, when they are counted, tabulated, or reported. In an effort to combat this problem, political parties often undertake their own monitoring, setting up a

presence at polling stations and observing or even involving themselves directly in the process of recording votes (e.g. parallel vote tabulations). Another especially effective approach has been reporting of results of exit polls in near-real time using cell phone-based tools, as was observed originally in the Ukraine in 2004. More generally election monitoring by independent observers from civil society, the media and international organizations has become commonplace (Hyde 2011; Kelley 2012). Social scientists have been adding to these efforts. For example they have evaluated methods aimed at reducing the manipulation of results, e.g., using communications technology to take photos of tabulations at polling stations (Callen, Long and Isaqzadeh 2011; Callen and Long 2013; Callen, Gibson, Jung and Long 2013).

Election monitoring and observation is often conducted in person. Yet in-person observation has certain inherent limitations. The monitors themselves, especially individuals from within a given country, may not be impartial. Those on the losing side have incentives to claim that results are fraudulent, whether or not this is true, in an effort to gain leverage or disrupt the process. Likewise those on the winning side have incentives to conclude that the elections were free and fair. Even if observers are independent their access to polling places may be restricted by the government or by local authorities. In addition observers may not be able to blanket an entire country for logistical, financial, political or safety reasons. Even where full access and coverage are feasible, the detailed information collected by observers is rarely made available publicly, especially in a comprehensive fashion at the level of electoral constituencies, let alone precincts and polling stations. Instead the information is generally compiled into summary reports, which may feature select anecdotal examples of malfeasance. These reports usually offer an overall judgment about whether an election was free and fair, which is not necessarily immune to politicization because of the stakes that may be involved. In-person observation can be a valuable input into assessments of electoral fraud, but rarely offers a comprehensive evaluation of individual contests such as those for specific parliamentary seats. Meanwhile

the prospect of scholars setting up on-the-ground experimental interventions to address fraud in elections around the world is unrealistic, since this would again require the necessary access, as well as massive personnel, financial and technological resources.

2 The Role for Election Forensics

As useful as any existing method may be, none meet all of the ideal criteria outlined in the previous section. In this regard new and developing methods in election forensics offer tools that can conceivably meet all of our ideal criteria. Scholars in the field of election forensics have been developing techniques designed to detect the presence of fraud or malfeasance in an election, and estimate its magnitude, based on characteristics of the reported results. Some of the election forensic methods adopt techniques similar to those developed by those seeking to detect financial fraud. One basic idea the methods share is that numbers manipulated by humans typically have patterns of digits that are unlikely to have arisen through a "natural" process—such as a free and fair election or normal commercial transactions. In essence, numeric manipulations (frauds) tend to leave distinctive traces. If election results have unnatural patterns, those anomalies should be detectable using statistical techniques.

Relative to existing approaches, election forensics methods have at least three key advantages. First, election forensics relies on reported election results, disaggregated to the level of electoral constituencies, precincts, and/or polling stations. These results are "objective" in the sense that they are used exactly as reported—forensic methods analyze the official record, which are the public basis of election outcomes. By contrast, the observations and judgments of individual election monitors, country experts, or assessment teams can be (or be accused of being) selective and subjective.

Second, election forensics allows for the systematic analysis of all reported votes from all contests in all localities—even those locations where observers do not have access for

political, security or logistical reasons.

Third, election forensics produces estimates of fraud that include information about the confidence of conclusions, as opposed to observations methods that lack these metrics beyond some vague expression of certainty vs. uncertainty, which may not be well founded.

At the same time, election forensics have three primary disadvantages. First, producing the statistics does require a fairly sophisticated knowledge of quantitative methods, as well as substantial computing power. The Electoral Forensics Toolkit presents one potential solution to overcome this challenge: experts perform the statistical work in-house and provide a report of the results that is accessible and intelligible to non-specialists.

Second, election forensics do not produce definitive proof of fraud—unlike direct observation of ballot box stuffing. Instead, the method produces probabilistic evidence of anomalies that are suggestive of fraud. This limitation need not fundamentally undermine the election forensics approach, especially given that other methods of election assessment are similarly constrained (again, short of tangible proof of malfeasance). Rather, the limitations of election forensics should caution against making overstated inferences or claiming that the approach can supplant all others, errors that we are conscious to emphasize as concerns and to avoid.

Third, the utility of election forensics methodology is limited if election results are not reported in a sufficiently detailed fashion. The method works best the more detailed and disaggregated the data are. The ideal is comprehensive data on turnout, valid and spoilt ballots, together with all votes for all parties and candidates down to each and every polling place (or even down to every ballot box), though the tools can also work with less detailed data. Election forensics are of limited use, however, if election results are provided only as a summary at a national level, or not reported at all (e.g., some countries only report who was elected, or the distribution of seats among the parties contesting the election). In these cases, assessment tools other than election forensics are more appropriate—and may be the only realistic options.

Fortunately, a vast majority of countries do provide subnational election results. In fact, even many countries that conduct elections in a context marked by severe restrictions on political mobilization and competition, with boycotts and/or accusations of fraud by opposition parties, still report subnational results. The availability of these detailed results allows election forensics to be married with complementary methods and tools, such as in-person monitoring. Election forensics enhance the value of more labor-intensive forms of monitoring, instead of being a wholesale substitute. Anomalies and other actual or suggestive evidence of malfeasance detected by one or more approaches can be combined in ways that are mutually reinforcing. For example, users might combine our estimates of anomalies with information from country officers or election observers, which may be reflected in datasets such as the Electoral Integrity Project, Kelly and Kolev's Quality of Elections Database or Lindberg's Elections and Democracy in Africa to produce a more complete, cross-validated picture of a given election.[1]

2.1 A Primer on Election Forensics Methods

Two assumptions underlie the use of election forensics. First, statistical analysis of suitably disaggregated election data can detect instances of illegitimate human manipulation. Second, patterns in data that typically result from such manipulation, as well as patterns of deviations from the distributions that are expected in the absence of manipulation, are more likely to represent evidence of fraud if found consistently across multiple statistical techniques.

A variety of methods have been used to evaluate election results as indicators of anomalies. Past research has focused on several kinds of data reported by government election authorities: voter turnout, votes received by various parties or candidates, and number of ballots deemed invalid. Some researchers analyze combinations of these reported

[1]Electoral Integrity Project: `http://www.electoralintegrityproject.com/`. Quality of Elections Database: `http://sites.duke.edu/kelley/data/`. Elections and Democracy in Africa: `http://www.clas.ufl.edu/users/sil/downloads.html`.

data. The research can go in one of two directions. One direction assumes what should happen in an election where anomalies do not occur. Another direction takes the complementary approach and assumes what should happen if manipulations are attempted. Different statistical methods may respond to different kinds of anomalies. When the various methods lead to similar statistical indications—e.g., indications that there is a high likelihood that there are anomalies in reported data—it adds strength to our conclusions (i.e., that election results have been manipulated).

The first approach to anomaly and fraud detection centers on the digits of the vote count values as indicators of anomalies. The first digits of aggregate vote totals (Cantu and Saiegh 2011), second significant digits (Pericchi and Torres 2011) and the last digits (Beber and Scacco 2012) have all been of interest. The second digit methods are based on the idea that nonanomalous vote counts follow the so-called second-digit Benford's Law-like (2BL) distribution—a particular distribution of numerical digits occurring from a natural process with each number 0-9 as differentially likely (Pericchi and Torres 2004; Mebane 2006). The last-digit method is based on the idea that unmanipulated vote counts have uniformly distributed 0-9 last digits. The digit-focused methods are controversial as fraud-detection devices (Carter Center 2005; Shikano and Mack 2009; López 2009; Deckert, Myagkov and Ordeshook 2011; Mebane 2011, 2014), with in particular Mebane (2013*a*, n.d.) claiming that the second digits of precinct vote counts may be produced by normal political processes such as strategic voting and mobilization that do not trace back to illegitimate human manipulation.

A second approach relies on analysis of turnout data. This approach focuses on departures from the expected shape of the distribution of voter turnout (Myagkov, Ordeshook and Shaikin 2009; Levin, Cohn, Ordeshook and Alvarez 2009) or on the relationship between turnout and the share of votes for the leading candidate (Buzin and Lubarev 2008). Also particular patterns in turnout and vote proportions have been identified because they facilitate coordination among election malefactors (Kalinin and

Mebane 2011; Mebane 2013*b*).

Third, invalid vote counts have also been useful in diagnosing possible irregularities (Mebane 2010). A paucity of invalid votes that match a high proportion of votes for the leading candidate can reveal that votes have been faked, possibly through careless ballot box stuffing.

A fourth approach was introduced as a simulation method by Klimek, Yegorov, Hanel and Thurner (2012) and turned into a statistical estimation method by Mebane (2015*a*) (see also Mebane, Egami, Klaver and Wall (2014); Mebane and Klaver (2015); Mebane and Wall (2015)). This method offers a sophisticated model of irregularities that uses data on turnout, valid ballots and the number of votes received by each party or candidate in election aggregation units (such as polling stations) to produce estimates of the occurrence and magnitude of extreme fraud and incremental fraud. The specifics of this approach will be described in further detail below.

2.2 Key Statistical Methods

We further define the statistical tools in each approach and then illustrate how the statistics could be used to diagnose the accuracy or inaccuracy of election outcomes. In describing the statistics we include the number of people voting when we refer to "vote counts." The proportion of those eligible to vote who do vote—a conventional notion of "turnout"—is treated similarly.

2.2.1 Digit and Turnout Distribution Approaches

Table 1 summarizes some of the statistics and tests employed by the digit and turnout approaches. These tests are implemented as part of the Election Forensics Toolkit. More detailed definitions of each of the statistics are in the Appendix.

Table 1: Distribution and Digit Tests

Test	Name	Definition	Value expected in the absence of fraud or strategic behavior
second-digit mean	2BL	to be compared to the mean value Benford's Law specifies	4.187
last-digit mean	LastC	to be compared to the mean value implied by uniformly distributed last digits	4.5
count last-digit 0/5 indicator mean	C05s	to be compared to the mean value implied by uniformly distributed last digits	0.2
percentage last-digit 0/5 indicator mean	P05s	to be compared to the mean value implied by uniformly distributed percentages	0.2
skewness	Skew	the extent to which a variable departs from a normal distribution by being asymmetric	0
kurtosis	Kurt	the extent to which a variable departs from a normal distribution by being spread out too much or not enough	3
unimodality test p-value	DipT	tests whether the distribution of a variable departs from unimodality	> 0.05

We also use other methods to try to diagnose whether and where frauds occur. These methods are also part of the Election Forensics Toolkit.

- Multimodal frauds simulation model. Klimek et al. (2012) introduce a simulation model in which the baseline assumption is that votes in an election with no fraud are produced through the interaction of processes whose effects can be summarized by two Normal distributions: one distribution for turnout proportions and another, independent distribution for the proportion of votes going to the "winner" (that is, the party with the most votes). Klimek et al. (2012) condition on the number of eligible voters. Klimek et al. (2012) assume that election fraud means that votes are added to the votes for the winner. Some votes are transferred to the winner from the

opposition, and some are transferred from nonvoters. The two kinds of election fraud refer to how many of the opposition and nonvoters' votes are shifted: with "incremental fraud" moderate proportions of the votes are shifted; with "extreme fraud" almost all of the votes are shifted. Klimek et al. (2012) have parameters that specify the probability that each unit experiences each type of election fraud: f_{i} is the probability of incremental fraud and f_{e} is the probability of extreme fraud. Other parameters fully describe bimodal and trimodal distributions that the model characterizes as being consequences of election frauds. In this Guide and in Mebane (2015*b*) we do not use the results from executing the simulation model, although such results are available from the Election Forensics Toolkit.

- Multimodal frauds likelihood model. Mebane (2015*a*) introduces a finite mixture likelihood model that takes its definitions of election frauds from the simulation model proposed by Klimek et al. (2012). Earlier versions of the model were reported in Mebane et al. (2014) and Mebane and Klaver (2015). In this Guide we do not focus on estimates of the frauds probabilities f_{i} (incremental fraud) and f_{e} (extreme fraud) that may be obtained for an entire election. Instead we emphasize (1) statistical tests for frauds and (2) estimates of the probability that each observed vote aggregation unit—e.g., each polling station—is fraudulent. The statistical tests (likelihood ratio tests) are tests of whether, for all the election-unit observations in a given "district," using the three-component mixture model that includes frauds improves the fit to the data compared to a model that excludes both of the fraud components (Mebane and Wall 2015). An electoral unit is the smallest unit at which we observe aggregated votes (for our countries the electoral units are either polling stations or, in Kenya, "wards"). A "district" is the set that encompasses all the electoral units in an election—it is the geographic unit at which election winners are determined. In the case of legislative elections, a district can be a constituency with a single representative, or a larger geographic area—such as a province or even an

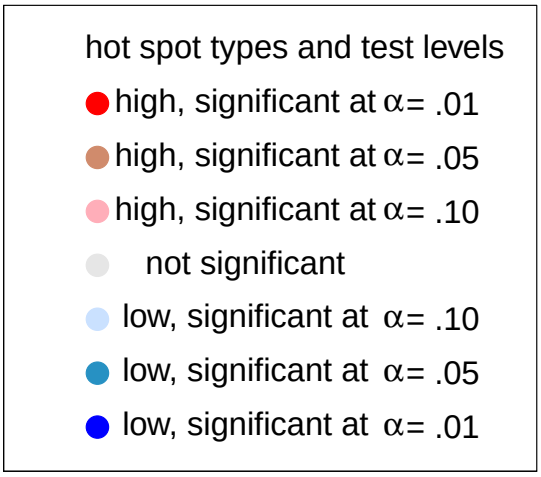
entire country—in which multiple seats are awarded using some proportional representation procedure. In the case of presidential elections, a district for purposes of the analysis is the entire country. The observation-level estimates of fraud probabilities are side-effects of the EM algorithm we use to estimate the finite mixture likelihood model (for details see Mebane and Wall (2015)).

- Geographic clustering tests. Indicators or phenomena that are geographically clustered are particularly noteworthy. Geographic clustering may show where there is cooperation or collaboration of the kind that occurs when election frauds occur. Geographic clustering may also suggest—to those with relevant substantive and local knowledge—other factors that could contribute to observed patterns in election results. These other factors can be related or unrelated to the possibility of fraud. For example a cluster might coincide with the home base of a political leader or an area in which the leader's political party or ethnic group is predominant (or in the minority).

  Getis-Ord $\mathbf{G_i}$: hotspots (Getis and Ord 1992; Ord and Getis 1995). This measures whether the mean of values geographically close to observation i differs from the overall mean. To test whether each G_i value is significantly larger or smaller than would be expected by chance, we use permutation test methods to estimate p-values. Details are specified in Mebane (2015*b*). The p-values are corrected for multiple testing using false discovery rate procedures (Benjamini and Hochberg 1995) for the test levels α shown in Figure 1 ($\alpha = .01$, $\alpha = .05$ and $\alpha = .1$).

  local Moran's $\mathbf{I_i}$: local spatial clustering (Anselin 1995). This measures whether the value at observation i differs from the mean of values geographically close to the observation i. To test whether each I_i value is significantly larger or smaller than would be expected by chance, we use permutation test methods to estimate p-values. Details are specified in Mebane (2015*b*). p-values are corrected for multiple testing using false discovery rate procedures (Benjamini and Hochberg 1995).

Figure 1: Hotspot Analysis Legend



Note: Significance levels refer to tests adjusted for the false discovery rate (Benjamini and Hochberg 1995). This figure displays the legend for subsequent hotspot maps used in this paper. Red colors show areas where local average scores are significantly above the overall average. Blue colors show areas where local average scores are significantly below the overall average.

The foregoing is not an exhaustive listing of all the methods that have been proposed to detect election frauds, but these are the methods on which we have focused.

In seeking to diagnose "where election frauds occur" our aim is to specify the particular geographic locations of the frauds. If data about the counts of voters and vote choices are adequate, then the finite mixture likelihood method allows us to estimate probabilities that frauds occur at each observation for which we have data. Other methods allow us to estimate where there are geographic clusters of indicators that may suggest that frauds occurred. The various kinds of measures relate to different aspects of the elections. The variety of measures can help us to address the most important challenge involved in election forensic analysis, namely, the difficulty of distinguishing effects of frauds from effects of strategic behavior such as strategic voting.

Some methods that may be sensitive to frauds are also sensitive to strategic behavior.

Extensive evidence shows that the second significant digits of precinct-level (polling station-level) vote counts respond to many different kinds of strategies. Measures of the two kinds of "frauds" in the finite mixture model have also been found to be related to measures of strategic behavior.[2] How other methods such as assessing whether last digits have a uniform distribution relate to strategic behavior has not yet been studied very extensively. The coincidence of both ambiguous measures and measures that may not be as ambiguous can help increase confidence in a diagnosis that frauds occur. But "signaling" patterns arise only given particular kinds of frauds, so the absence of such patterns is not evidence that there are no frauds.

Beyond urging caution about strategic behavior potentially being confused for frauds and reminding the reader about the possibility of other reasons for false positive indications of frauds, we repeat a final overall caveat: some fraudulent activities may not be detected using these methods. These techniques can also produce false negatives. The reliability of the techniques is a matter we are investigating in continuing research.

2.3 Examples of Election Forensics Analysis

To illustrate the methods for election forensics that we have studied, we give examples from two countries: the national election in South Africa in 2014 and the legislative election in Bangladesh in 2001. Fuller discussions of these elections (as well as of elections from the other six countries within the scope of subaward #DFG-10-APS-UM) appear in Mebane (2015*b*).[3] Mebane (2015*b*) also describes the sources and methods used to produce all the data. Analysis regarding all the elections may also be accessed through the Election Forensics Toolkit (`http://electionforensics.ddns.net:3838/EFT_USAID/`) (see also the "Forensics Toolkit" button at `http://www.electiondataarchive.org/forensics.html`).

[2]The values found using Klimek et al. (2012)'s original similation approach are also related to measures of strategic behavior.

[3]The elections we report on in Mebane (2015*b*) occur in Afghanistan 2014 (presidential election, first and runoff rounds), Albania 2013 (legislative election); Bangladesh 2001 (legislative election), Cambodia 2013 (legislative election), Kenya 2013 (presidential election), Libya 2014 (legislative election), South Africa 2014 (legislative election) and Uganda 2006 (presidential election).

These two examples illustrate what can be done when good, highly disaggregated data are available regarding eligible voters, vote choices and geography.[4] Organizing such data can require significant effort, as the Appendix in Mebane (2015*b*) describes. As will be apparent, the effort can be rewarded by the ability to reach extremely precise conclusions. It is possible to estimate for each observation—for each polling station, in the two cases we examine here—the probability that "frauds" occur. We can identify in a geographically precise way where "frauds" occur.

But even when data are very good, as in the cases of Bangladesh 2001 and South Africa 2014, we face the persistent dilemma that is the core challenge for election forensics: can statistically informed methods distinguish effects of fraudulent activites from effects of strategic behavior or of "normal politics" (Mebane 2013*a*, n.d.; Mebane and Klaver 2015). In more focused case studies of these or other elections, additional information from covariates, interviews, first-hand observations and other sources would potentially support more definitive judgments about a diagnosis that frauds occurred and where they occurred. Nuanced and difficult issues arise when using such information. For example, if covariates are related to frauds measures—or if the frauds measures dissipate when covariates are introduced—does that mean that the frauds measures are masquerading for normal political configurations the covariates help capture or that the covariates are helping pinpoint the conditions in which the frauds occur? If claims are made that strategies are being used, are those claims cover stories for frauds that are being committed?

It is also important to recognize that some of the methods used here are in early stages of their development. This is particularly true of the finite mixture likelihood model of frauds (Mebane 2015*a*; Mebane and Wall 2015). Future developments of these technologies may well change the impression they convey of when and where frauds and strategic actions occur.

[4]Mebane (2015*b*) discusses the potential for using data at higher levels of aggregation than polling station-level data. Data used in Mebane (2015*b*) from Kenya are at the "ward" level, not the polling station level. Bangladesh 2001 has enough districts (constituencies) to warrant applying our statistical methods to the constituency-level aggregations there.

2.3.1 South Africa 2014

In South Africa the members of parliament are elected using a party-list proportional representation system with voting at two levels, with two separate ballot papers per voter. Of the 400 seats in the parliament, 200 are apportioned using "National" ballot party lists, with votes considered on a nationwide level. The proportional breakdown of seats is calculated on a percentage-point basis—despite there being more than 100 seats contested, a party cannot win seats with less than 1% of the vote, and seats are not awarded for fractions of a percentage point. The other 200 seats are elected from "Provincial" ballot party lists, also by proportional representation, from nine multi-member districts matching the nine South African provinces. District magnitude in these districts ranges from four to 43 seats.[5]

Table 2 shows test statistics computed using polling station observations for the National component of the parliamentary elections in South Africa in 2014.[6] Results are shown for "Turnout," which is the number of voters voting, and for three parties: African National Congress; Democratic Alliance; and Economic Freedom Fighters. The table presents results for measures of digit distributions and of other features of the counts or of proportions computed using the counts. For the 2BL, LastC, P05s, C05s, Skew and Kurt statistics, the table shows the point estimate and a 95% confidence interval estimated using nonparametric bootstrap methods.[7] For DipT we present the p-value from a test of the unimodality hypothesis (Hartigan and Hartigan 1985).[8]

For turnout and for all three of the parties shown in Table 2, some estimates and tests deviate from what might be expected in the absence of frauds. Statistics that deviate from

[5]Sources: `http://www.ipu.org/parline-e/reports/2291_B.htm`, `http://www.elections.org.za/content/Elections/Laws-and-Regulations-Elections/`.

[6]All the statistics in tables like this throughout this Guide are computed using the Election Forensics Toolkit.

[7]We use the `boot` package (Davison and Hinkley 1997; Canty and Ripley 2015) of **R** (R Development Core Team 2005) to compute "basic" confidence intervals.

[8]We use the `diptest` package (Maechler 2013) of **R** (R Development Core Team 2005) to compute p-values using linear interpolation.

Table 2: Distribution and Digit Tests for National Votes, South Africa 2014

Level	Name	2BL	LastC	P05s	C05s
National	Turnout	4.229	4.466	0.236	0.2
		(4.191, 4.263)	(4.428, 4.507)	(0.23, 0.241)	(0.195, 0.205)
National	ANC	4.161	4.465	0.198	0.208
		(4.123, 4.199)	(4.426, 4.504)	(0.194, 0.204)	(0.203, 0.213)
National	Dem Al	4.022	4.17	0.177	0.21
		(3.978, 4.066)	(4.134, 4.206)	(0.172, 0.182)	(0.205, 0.216)
National	EFF	4.026	4.069	0.223	0.22
		(3.981, 4.075)	(4.03, 4.107)	(0.217, 0.229)	(0.214, 0.225)

Level	Name	Skew	Kurt	DipT	Obs
National	Turnout	−0.039	3.918	1	22263
		(−0.101, 0.021)	(3.709, 4.121)	−−	
National	ANC	-1.166	3.206	0	22260
		(−1.193, − 1.14)	(3.123, 3.286)	−−	
National	Dem Al	1.942	5.436	0	22260
		(1.897, 1.985)	(5.231, 5.618)	−−	
National	EFF	1.854	8.473	0	22260
		(1.734, 1.96)	(7.357, 9.457)	−−	

Note: "2BL," second-digit mean; "LastC," last-digit mean; "C05s," mean of variable indicating whether the last digit of the vote count is zero or five; "P05s," mean of variable indicating whether the last digit of the rounded percentage of votes for the referent party or candidate is zero or five; "Skew," skewness; "Kurt," kurtosis; "DipT," p-value from test of unimodality; "Obs," number of polling station observations. Values in parentheses are nonparametric bootstrap confidence intervals.
ANC, African National Congress. Dem Al, Democratic Alliance. EFF, Economic Freedom Fighters.

the values they are expected to have in the absence of fraud are highlighted in red. Some statistics have stronger implications for the presence of frauds than others. For turnout counts (i.e., the number of votes cast at each polling station) no one has stated a particular expectation for what the distribution of the second significant digits should be, so the fact that the digits have a mean (2BL) that is significantly greater than 4.187 is not especially meaningful. On the other hand, it is difficult to think of a benign reason why the last digits of the rounded turnout percentages should have an excess of values that are zero or

five (P05s).[9] So the fact that the P05s statistic is significantly greater than its nominal value of 0.2 suggests that some kind of frauds occurred. Notably the P05s statistic for turnout is significantly high even while the last digit mean of the turnout counts (LastC) does not differ significantly from its nominal value of 4.5. Neither is the mean of the 0-5 indicator variable for the last digits of the turnout counts (C05s) significantly different from its nominal value of 0.2. Something like frauds that involve "signaling"—which means that dispersed "agents" are trying to claim credit by manipulating turnout—seems to be occurring.

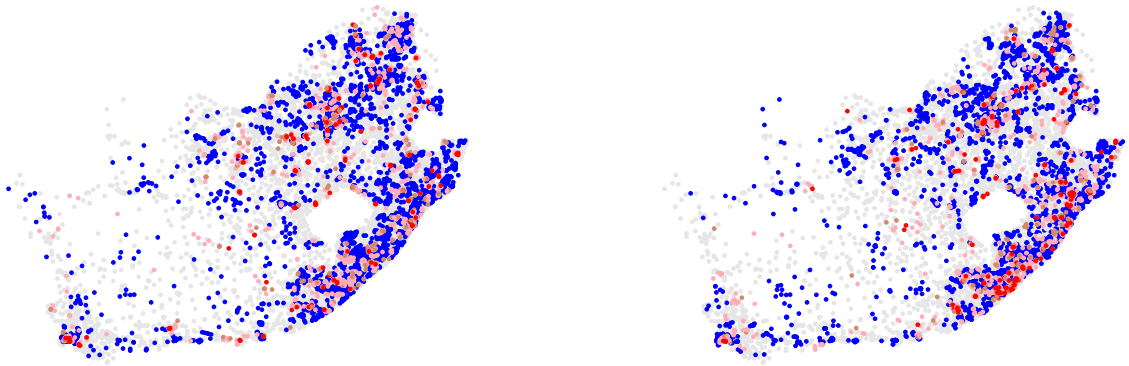
For two of the parties for which results are shown in Table 2 every statistic differs significantly from the values that might be expected in the absence of frauds. The 2BL, LastC and P05s statistics for the ANC do not differ significantly from their nominal "no-fraud" values, but all the statistics for Democratic Alliance and EFF do differ.[10] The 2BL statistics for these parties are all ambiguous, because patterns in which the second-digit mean is significantly below 4.187 arise frequently as a consquence of strategic behavior (Mebane 2013*a*). The second digit means provoked by strategic voting in such situations are comparable in magnitude to the values observed here for Democratic Alliance and EFF. The fact that skewness (Skew) and kurtosis (Kurt) differ significantly from what one would observe if the vote proportion distributions were Normal is not surprising given that the proportions have multimodal distributions (DipT is very small).

For South Africa in 2014 we have geolocation coordinates for each polling station, so we can estimate measures of geographic clustering. Figure 2 shows the results from estimating and assessing the significance of the G_i statistic computed using the vote count last-digit 0-5 indicator variable for two of the parties, namely ANC and Democratic Alliance. Despite LastC in Table 2 not differing significantly from 4.5 for the ANC and C05s

[9]The turnout percentage is the number of valid votes cast divided by the number of eligible voters.

[10]The P05s statistics for parties are estimated using the parties' proportions of all valid votes that were cast for a party. The denominator excludes spoiled, blank or otherwise invalid ballots, where it is possible to identify those. In situations where vote counts for write-in candidates are reported, those votes are counted as valid votes.

Figure 2: Last-digit 0-5 Indicator Hotspot Analysis, South Africa 2014

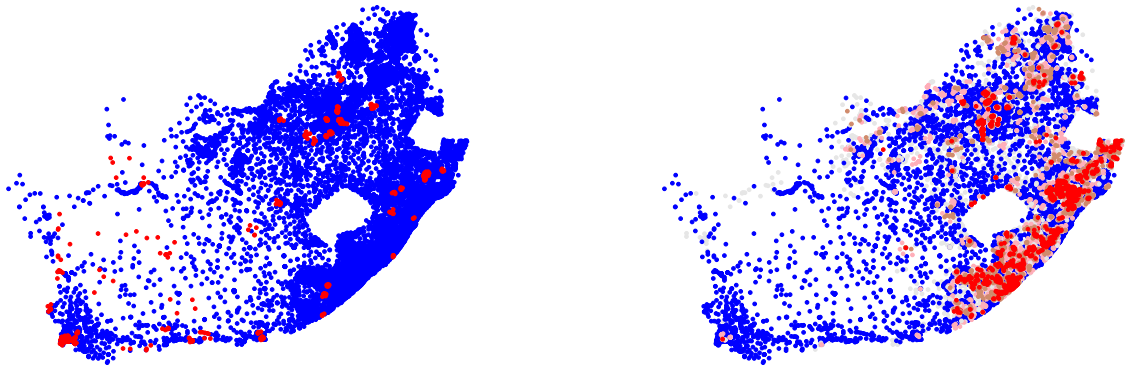


(a) ANC (b) Democratic Allliance

Note: polling centers, last-digit 0/5 indicator hotspot analysis using Getis-Ord G_i.

differing significantly but only slightly from 0.2, Figure 2(a) shows there is significant geographic clustering in the vote count last-digit indicator variable for that party. While for the Democratic Alliance both C05s and LastC in Table 2 do differ significantly from their nominal values—although C05s does not differ by much (the lower bound of the 95% confidence interval is .205)—the degree of geographic clustering in the 0-5 indicator variable in Figure 2(b) appears to be only slightly greater than that for the ANC in Figure 2(a). Red spots show where there are clusters of atypically high values—that is, places where the local mean of the indicator variable is significantly greater than the overall mean value. These are more frequent and themselves somewhat more clumped together for the Democratic Alliance vote counts than for the ANC vote counts. In Figure 3 we see that the disparity between parties in the pattern of high-valued clusters is even greater for the indicator of whether the last digits of the parties' rounded turnout percentages are zero or five. For the ANC there are relatively few clusters of significantly high values. For the Democratic Alliance there are many more. Note that the evidence in Figures 2 and 3 that there is geographic clustering does not tell us which parties benefited or were harmed by whatever activities produced that clustering.

Figure 3: Percentage 0-5 Indicator Hotspot Analysis, South Africa 2014



(a) ANC (b) Democratic Allliance

Note: polling centers, rounded percentage last-digit 0/5 indicator hotspot analysis using Getis-Ord G_i.

Estimating the finite mixture likelihood model of frauds can tell us whether there were frauds that benefited the party that received the most votes. In the South Africa 2014 national election, that party is the ANC. The additional modes that the finite mixture model says are "frauds" are not necessarily produced by fraudulent activities, but the appearance of geographic clustering in the last-digit 0-5 indicator variables has already raised suspicions. To the extent that patterns based on the finite mixture model estimates appear to confirm the appearance of genuine frauds, we have greater confidence in a conclusion not only that frauds occurred but also about where frauds occurred. For the South Africa election, fortunately, we have sufficient data to accomplish this full diagnostic plan. We have data that shows the number of eligible voters (called "REGISTERED VOTERS" in the vote data) at each polling station. With that variable as well as the vote counts for each party, we can estimate the finite mixture frauds model.

The first point in the analysis is to test in a statistically rigorous way for the presence of frauds. A likelihood ratio test shows that using the two fraud components in the finite mixture model significantly improves the model's fit to the data, compared to a model from

which the fraud components are excluded.[11] We have not tried to assess whether a model that includes only one of the fraud components is significantly worse than the model that includes both of them. The algorithm used to estimate the finite mixture model automatically drops fraud components when the probabilities f_{i} or f_{e} estimated for them become very small.[12] Neither fraud component was dropped with the data from the South Africa 2014 national election.

Second, the finite mixture model produces an estimate for each polling station of the probability that the vote data from that polling station are part of the no-fraud distribution, the incremental fraud distribution or the extreme fraud distribution. In all cases, according to the model, frauds benefit the party that received the most votes in the election.[13] The question is how much the frauds benefit that party. The finite mixture model includes parameters that characterize the magnitude of frauds across the entirety of each election and it may be possible to use these election-level magnitude estimates to approximate the magnitude of frauds in each polling station. We do not attempt to do that here. So we can say which type of fraud a polling station is likely to have experienced but not how many votes the fraud caused to be misattributed.

We do not report the frauds probabilities associated with each of the 20,185 polling stations used to estimate the finite mixture model with the national election data, but we do show what happens when we check whether those polling station-level frauds probabilities are geographically clustered. First we determine the overall average values to which the local averages are being compared to identify clustering patterns. Note that the overall average value of the extreme fraud probabilities is several orders of magnitude lower than that of the incremental fraud probabilities. Estimates of the election fraud

[11] For details about the test results see Mebane (2015*b*).

[12] A fraud component is dropped when its probability f_{i} or f_{e} becomes less than 10^{-9} in the iterative estimation algorithm. In that case the small probability is set to zero. See Mebane (2015*a*) or Mebane and Wall (2015) for details.

[13] Allowing only one party to benefit from frauds is a limitation of the Klimek et al. (2012) conception, but it is what we have to work with at the moment. Mebane et al. (2014) outline this limitation and other limitations of the Klimek et al. (2012) concept.

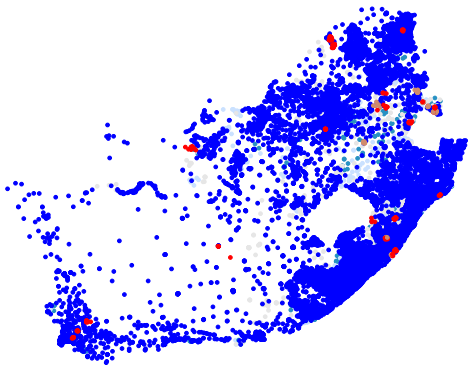
probabilities, f_{i} and f_{e}, are averages of the observation-level probabilities. For the national election these values are $\hat{f}_{\mathrm{i}} = .074$ and $\hat{f}_{\mathrm{e}} = .0000087$. To put the national election results in some context, we also consider the fraud probabilities estimated for the provincial election votes.[14] For the provincial elections the frauds are almost all incremental frauds: $\hat{f}_{\mathrm{e}} = 0$ in all provinces except Limpopo.[15] The overall average for the incremental fraud probabilities in the provincial election data is 0.159. Overall, frauds are more likely in the provincial elections than in the national election, even though extreme frauds occur more often—but still extremely rarely—in the national election.

Figure 4 displays clustering estimates using the G_i statistic computed using the estimated frauds probabilities. Figures 4(a,b) show results using extreme and incremental frauds probabilities estimated for the national votes. Figure 4(c) shows results using incremental fraud probabilities estimated for the provincial votes. Right away it is apparent that local clusterings of high values of the extreme fraud probabilities occur much less frequently than do clusterings of high values of the incremental fraud probabilities. But comparing Figures 4(a) and 4(b) shows that some of the areas in which there are clusters of high extreme fraud probabilities are also areas in which there are clusters of high incremental fraud probabilities. Some of these areas also show clusters of high local means for the 0-5 indicator variables in Figures 2 and 3. Those are very likely areas in which genuine frauds occurred, and in the national election these are frauds that benefit the ANC (the ANC is most often but not always the leading party in the provincial elections). Those same areas also exhibit clusters in which there are high incremental fraud probabilities in the provincial elections.

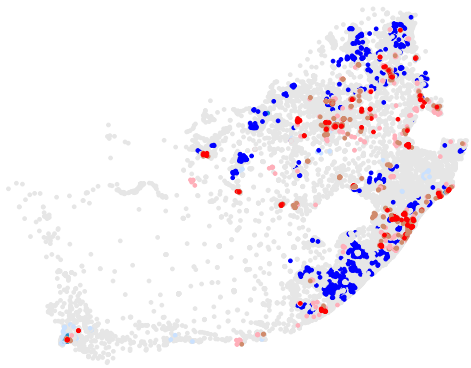
[14]The finite mixture model is estimated separately in each province. In each case likelihood ratio tests show the frauds components are significant.

[15]In Limpopo $\hat{f}_{\mathrm{e}} = 0.00000000775$. $\hat{f}_{\mathrm{i}}$ values for each province are as follows: Eastern Cape, .118; Kwazulu-Natal, .182; Free State, .0452; Gauteng, .112; Mpumalanga, .180; Limpopo, .179; Northern Cape, .210; North West, .199; Western Cape, .323.

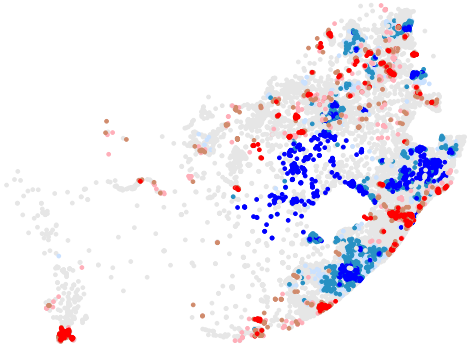
Figure 4: Hotspot Analysis, Fraud Probabilities, South Africa 2014



(a) national votes extreme fraud



(b) national votes incremental fraud



(c) provincial votes incremental fraud

Note: polling center fraud probability hotspot analysis using Getis-Ord G_i.

2.3.2 Bangladesh 2001

The Bangladesh 2001 election features separate contests in 300 single-member districts using plurality voting rules,[16] and we have vote data at the polling station level for 299 of those districts.[17] Controversially (Centre for Research and Information 2002), the Bangladesh National Party (BNP) won in a majority of the districts. We estimate the test statistics separately for each district. Figures 5 and 6 summarize the estimates and test

[16]Sources: `http://bdlaws.minlaw.gov.bd/pdf_part.php?id=367`, `http://www.ipu.org/parline/reports/2023_B.htm`, `http://www.ecs.gov.bd/MenuExternalFilesEng/378.pdf`.

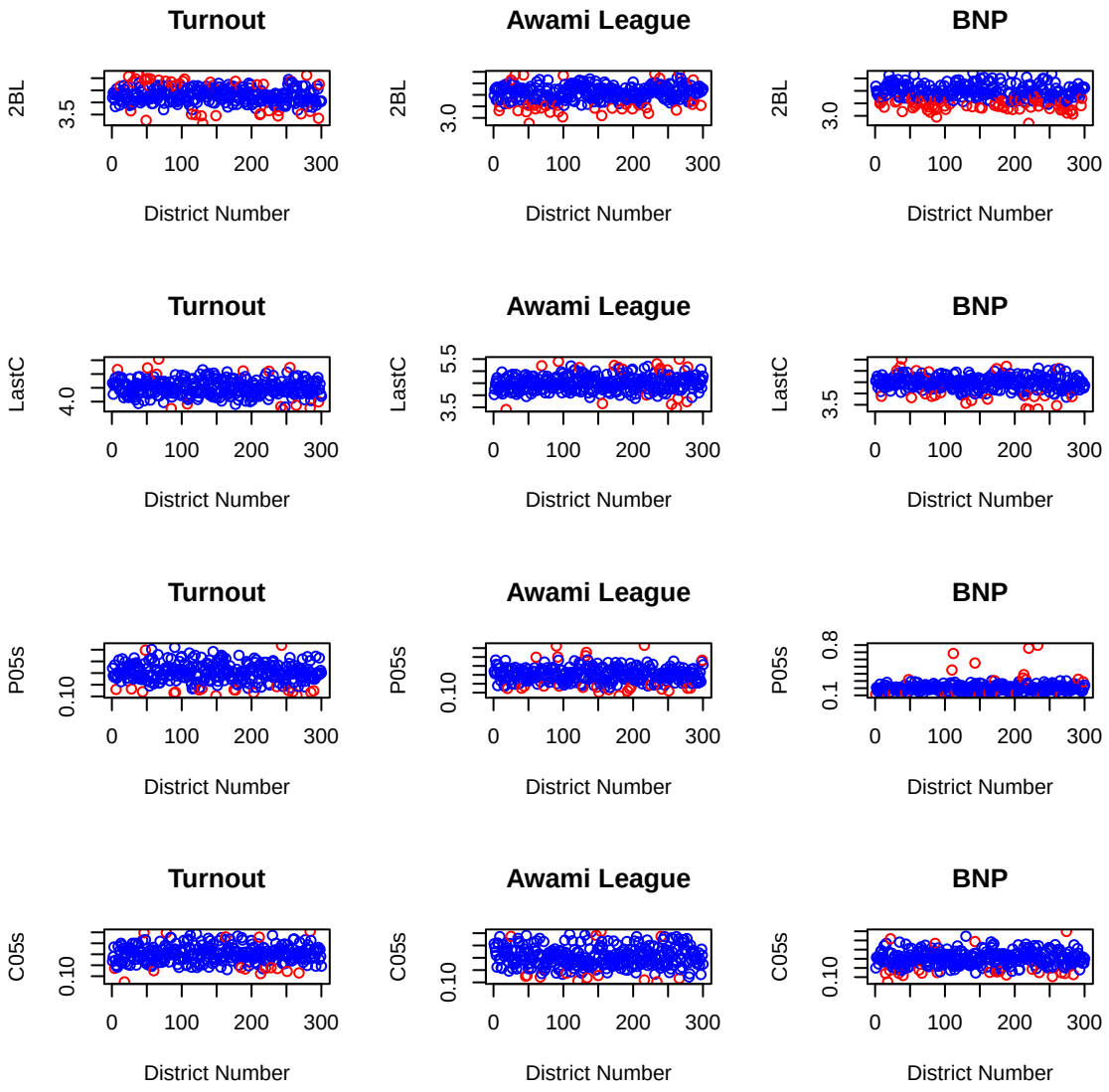
[17]Data for one district are missing in the raw data we have.

results in a graphical format. Details may be seen in tabular form in Mebane (2015*b*) and via the Election Forensics Toolkit. In each graph in Figures 5 and 6 we plot circles to show the value for each statistic or test for each district. The circle is blue if the confidence interval for the statistic includes the value we expect to see in the absence of fraud, otherwise the circle is red. For DipT a circle is red if the p-value is less than .05.

There is a lot of red in both figures, but that does not imply that there is genuinely a lot of fraud. The main problem is that both strategic voting and election frauds probably occur, and the statistics can respond to both. Given the single-member districts and plurality voting rules used in the election, we can expect that wasted vote strategies as analyzed by Cox (1994, 1997) occur: some voters choose their second-most preferred party instead of their most preferred party because they think their most preferred party has less support in their district. In this case voters switch their own votes between parties, most often to the benefit of the parties that finish in the top two places in each district. But there are also likely frauds—many problems were observed in the election (European Union 2001) and frauds were alleged (Centre for Research and Information 2002). Probably the frauds also tended to benefit the leading parties.

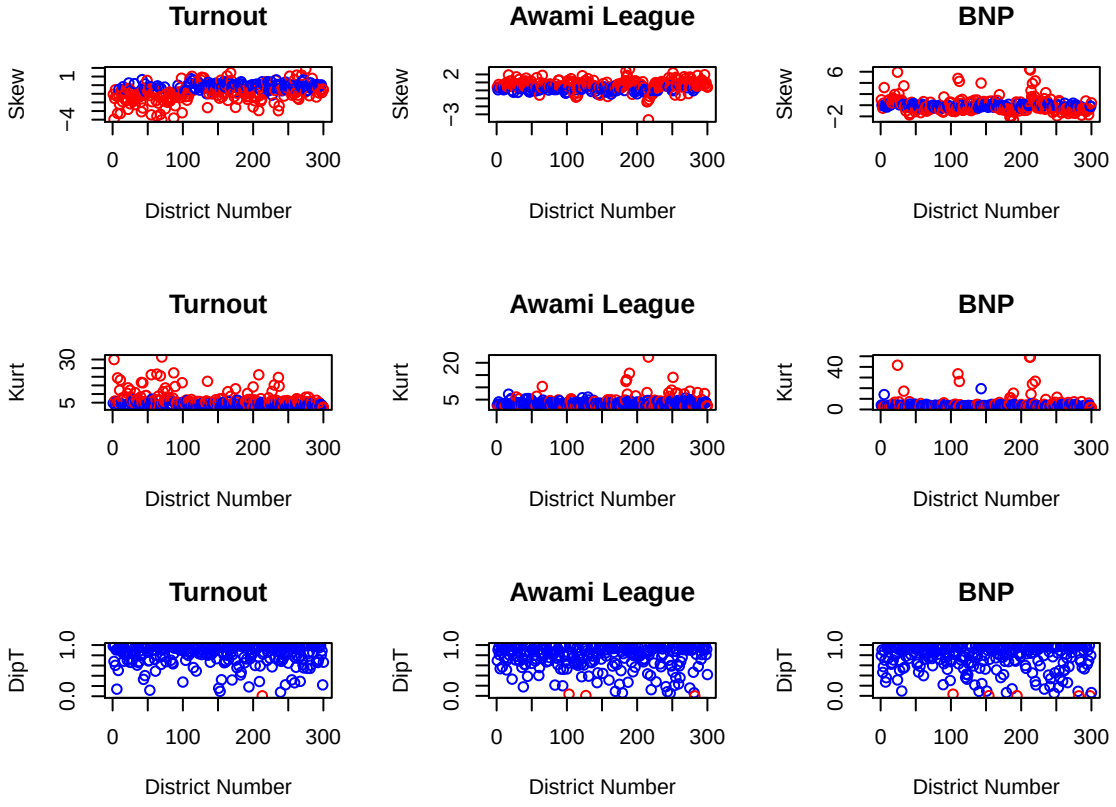
Almost certainly ambiguous in this regard are the results in Figure 5 for the 2BL statistic. Second-digit means computed from votes counted for the Awami League are often significantly lower than 4.187 but also sometimes significantly higher. Second-digit means computed from votes counted for the BNP are very often significantly lower than 4.187. The pattern in which the second-digit mean is low is frequently observed in situations in which there are multiple competetive parties running in a district with plurality rules and voters who act strategically (Mebane 2013*a*). A mix of significantly too-high and too-low values is harder to explain as due to strategic voting. The 2BL findings in Figure 5 strongly suggest that both strategies and frauds are at work. Later we examine conditional distributions of the second-digit means (see Figure 11), which provide further evidence regarding the balance between strategies and frauds in this election.

Figure 5: Distribution and Digit Tests by District, Bangladesh 2001



Note: statistics and tests based on polling station observations. "2BL," second-digit mean; "LastC," last-digit mean; "C05s," mean of variable indicating whether the last digit of the vote count is zero or five; "P05s," mean of variable indicating whether the last digit of the rounded percentage of votes for the referent party or candidate is zero or five.

Figure 6: Distribution and Digit Tests by District, Bangladesh 2001



Note: statistics and tests based on polling station observations. "Skew," skewness; "Kurt," kurtosis; "DipT," p-value from test of unimodality; "Obs," number of polling station observations.

The tests in Figure 5 that focus on the last digits of vote counts (LastC and C05s) or of turnout or vote proportions (P05s) show a mix of results that are either significantly too large or significantly too small. Digits that are frequently both too often zero or five and also, in other places, too rarely zero or five are difficult to square with "signaling" based on "agents" trying to claim credit for their efforts to commit frauds. Perhaps the patterns of geographic clustering among the 0-5 indicator variables will help us understand what is happening. Before considering explicit geographic clustering, however, it is interesting that the P05s statistic exhibits several occurrences of very high values for the BNP for nearby

district numbers (near 110 and near 220). Perhaps those districts are strongholds for the BNP, and perhaps they are geographically close to one another as well.

The skewness and kurtosis statistics in Figure 6 show that the distributions of turnout proportions and of vote shares are very often not Normal in the districts, yet the DipT p-values rarely give evidence that rejects the hypothesis that the distributions are unimodal. Such a combination of results is difficult to interpret. We turn to the finite mixture frauds model to get a clearer picture.

Because we have data that shows the number of eligible voters (called "Registration" in the vote data) at each polling station, we can estimate the finite mixture frauds model. That is a better tool for diagnosing potential frauds than merely examining skewness and kurtosis (Klimek et al. 2012).

We estimate the finite mixture frauds model separately in each district. Likelihood ratio tests show that including the two fraud components in the model significantly improves the model's fit to the data in 166 of the 299 districts. The 133 districts for which the finite mixture model gives no evidence of significant frauds are listed in Table 3. The DipT p-values shown in Figure 6 do not produce significant results for anywhere close to 166 districts. The sample sizes in the districts may be too small to trigger DipT but apparently are large enough that the finite mixture model detects additional modes, as parameterized by that model.[18]

Figures 7 and 8 display geographic clustering estimates using the G_i statistic computed using the estimated frauds probabilities. Figure 7 shows results using extreme fraud probabilities and Figure 8 shows results using incremental fraud probabilities. To compute G_i we first compute the average of the polling station frauds probabilities for each "Union," which is the smallest administrative unit for which we have boundaries in an ESRI shapefile.[19] G_i is computed and the statistical significance of the G_i values is assessed using these averages for each Union. The overall mean of the Union average probability

[18]For only 134 of the districts are there more than 100 polling station observations in the district.
[19]For information about the shapefiles see Mebane (2015*b*).

Table 3: Districts without Significant Election Frauds Components

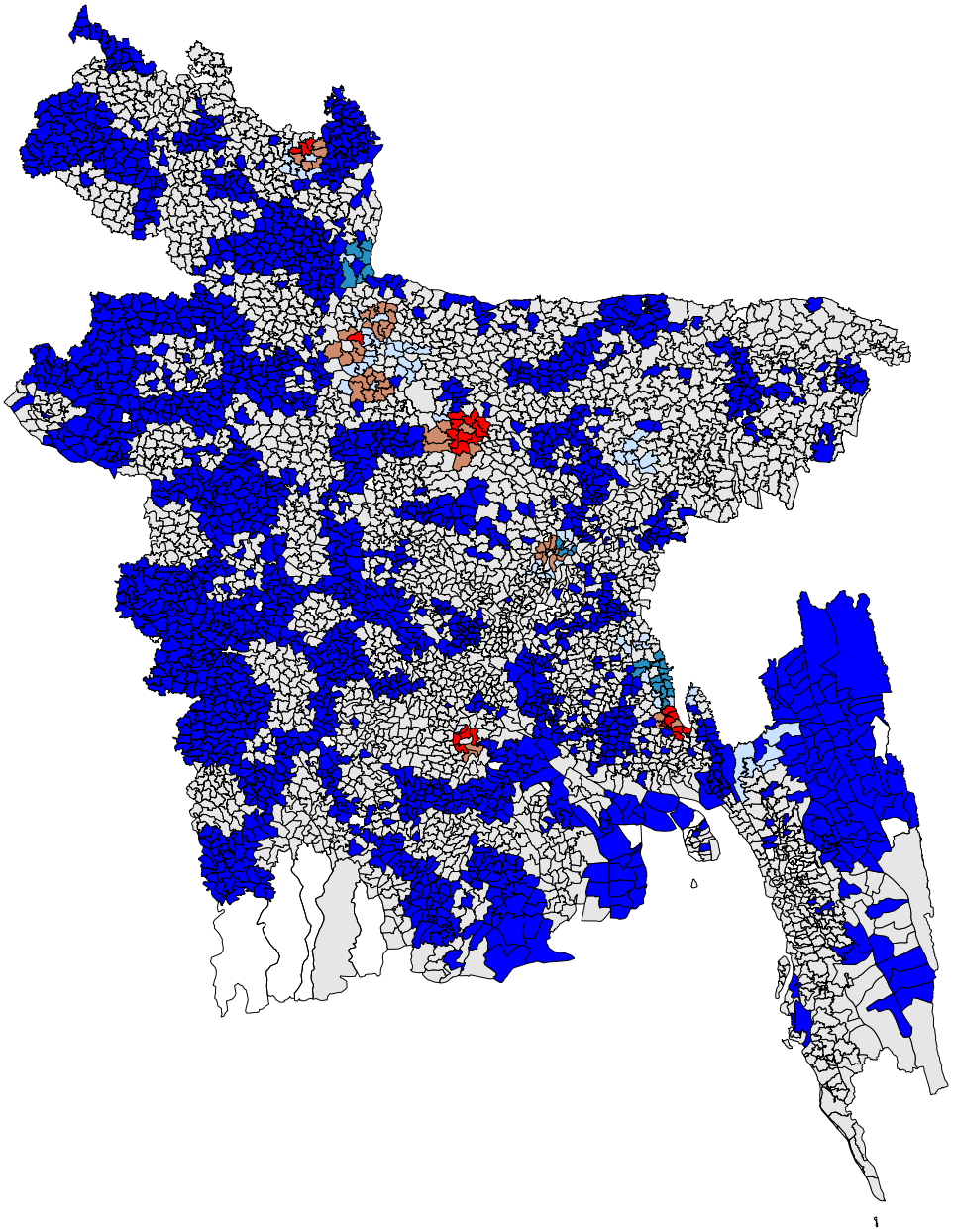
Country	Year	Count	"District"
Bangladesh	2001	133	3, 5, 6, 8, 10, 12, 13, 14, 15, 20, 22, 24, 30, 33, 34, 35, 36, 37, 38, 39, 41, 44, 45, 46, 47, 49, 51, 52, 53, 54, 56, 57, 58, 59, 60, 61, 62, 64, 65, 67, 73, 74, 75, 76, 77, 79, 80, 81, 82, 83, 84, 85, 87, 88, 90, 91, 92, 96, 100, 102, 103, 104, 105, 106, 107, 108, 109, 111, 115, 125, 129, 132, 134, 135, 136, 138, 139, 142, 150, 151, 153, 156, 157, 158, 159, 160, 167, 170, 171, 172, 173, 174, 176, 177, 180, 181, 186, 189, 191, 192, 194, 195, 197, 198, 204, 206, 207, 209, 212, 215, 216, 231, 234, 240, 243, 245, 251, 256, 260, 263, 264, 265, 271, 274, 279, 280, 285, 288, 290, 292, 293, 295

Note: "District" identifies election areas for which fraud components are not statistically significant according to a likelihood ratio test with false discovery rate correction across all data from Albania, Bangladesh, Kenya, South Africa (National and Provincial) and Uganda. Numbers shown for Bangladesh are legislative district numbers. Fraud components are significant for all other "districts," including all other legislative districts, provinces and whole countries.

values, to which local means are being compared to identify clusters, is 0.000200 for extreme fraud and 0.0435 for incremental fraud. As usual, the average value of the extreme fraud probabilities is orders of magnitude lower than the average value of the incremental fraud probabilities.

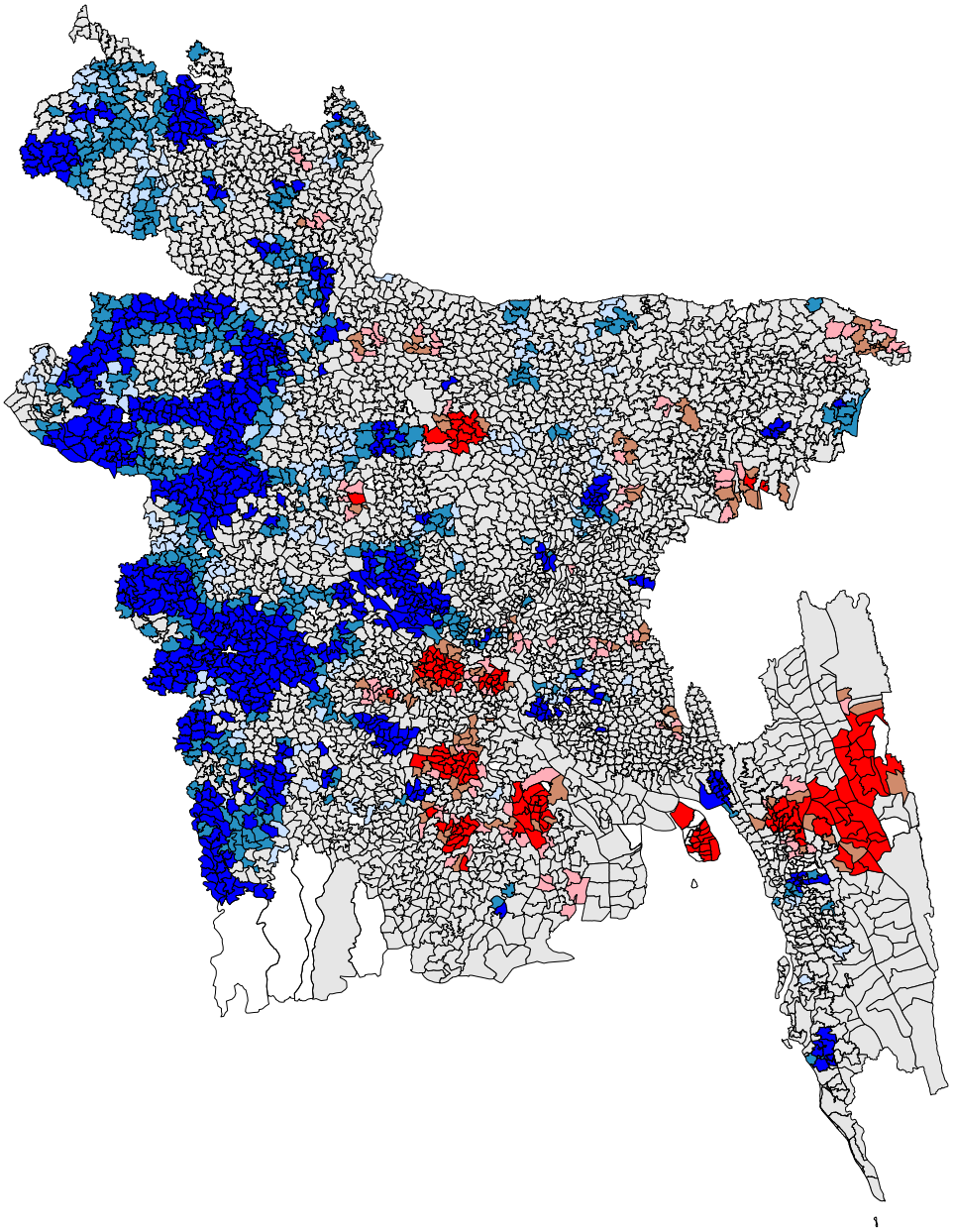
The frauds probabilities do exhibit significant geographic clustering. In Figure 7 there are several clusters with extreme fraud probabilities that are significantly greater than the overall average. Figure 8 shows that clusters with above average incremental fraud probabilities occur even more frequently. Most of the areas in which clusters of extreme fraud probabilities occur are also areas where clusters of incremental fraud probabilities occur. The key question is: Are the areas that exhibit significant geographic clustering of values of one or both types of frauds probabilities places where frauds genuinely occur?

Figure 7: Hotspot Analysis, Extreme Fraud Probabilities, Bangladesh 2001



Note: polling center fraud probability Union average hotspot analysis using Getis-Ord G_i.

Figure 8: Hotspot Analysis, Incremental Fraud Probabilities, Bangladesh 2001

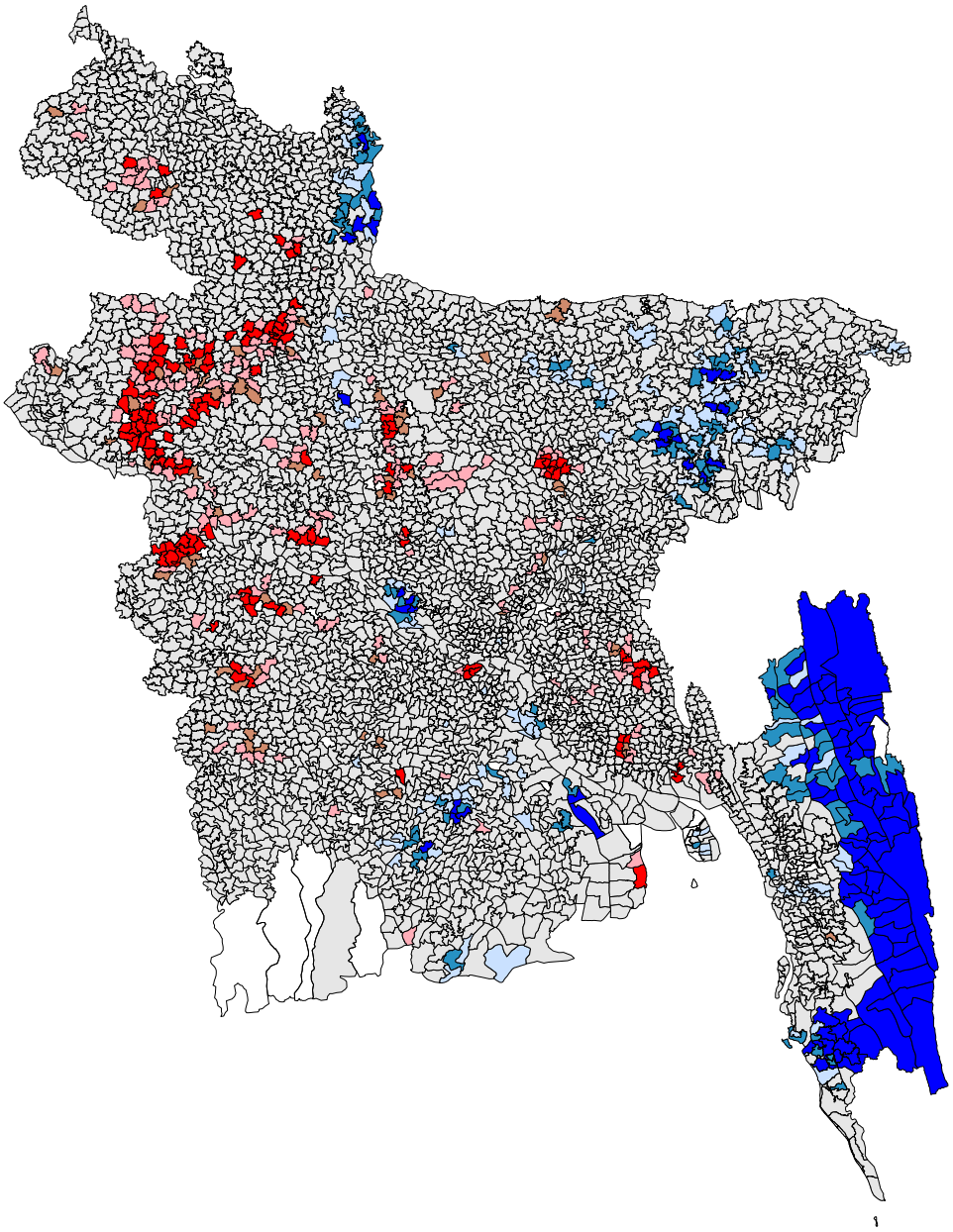


Note: polling center fraud probability Union average hotspot analysis using Getis-Ord G_i.

To address whether the apparent frauds are genuine frauds we examine whether the digit-based variables are geographically clustered and, if so, whether the clusters in the digit-based variables coincide with the clusters in the frauds probabilities. Probably the most informative variable to consider is the binary variable that indicates whether the last digit of the rounded percentages for each party is zero or five (i.e., the variable averaged to produce the P05s statistic), because a high mean for that variable is related to "signaling" ideas in which "agents" try to claim credit for the fraudulent activities they undertake. Of course "signaling" is not the only potential reason for distortions in the distribution of the indicator variable. Anything that causes the vote counts to go awry could impact the percentages.

Figures 9 and 10 show there is significant geographic clustering in the vote percentage 0-5 indicator variable no matter whether it is computed using vote percentages for the BNP or for the Awami League. Interestingly, many areas where there are clusters of significantly high values for one party are also areas where there are clusters of significantly high values for the other party (for example, in areas in the northwest). Likewise many areas with clusters of significantly low values are the same for both parties (to the east). In the central part of the country is where we see a few overlaps between areas with clusters of significantly high frauds probabilities and clusters of high averages of the indicator variable. Clusters for the BNP percentage indicator variable (Figure 9) overlap with a few of the high incremental fraud clusters (Figure 8; in the middle) but not so much the high extreme fraud clusters. Clusters for the Awami League indicator variable (Figure 10) overlap with a few of the high extreme fraud clusters (Figure 7; in the north, in the middle and to the south) and with a few of the high incremental fraud clusters (in the middle).

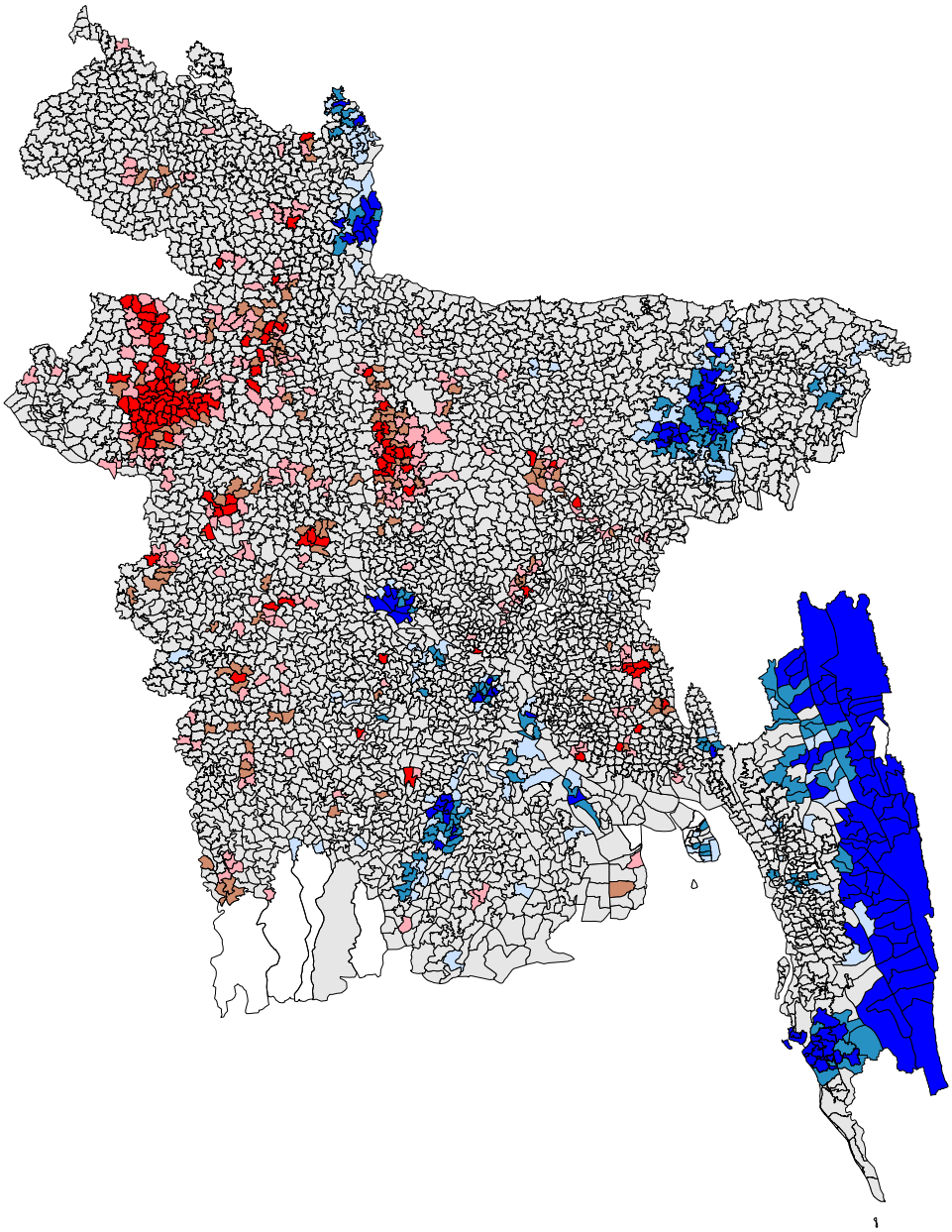
Figure 9: Hotspot Analysis, Percentage 0-5 Indicator, Bangladesh 2001, BNP



BNP

Note: rounded percentage last-digit 0/5 significant digit Union average hotspot analysis using Getis-Ord G_i.

Figure 10: Hotspot Analysis, Percentage 0-5 Indicator, Bangladesh 2001, Awami League



Awami League

Note: rounded percentage last-digit 0/5 significant digit Union average hotspot analysis using Getis-Ord G_i.

For the other variables that are based on the last digits of the vote counts significant geographic clusters are only sporadically observed—a few with significantly high values and a few with significantly low values. In a couple of instances the clusters for these variables overlap with clusters of significantly high values for the frauds probabilities, but on the whole they do not add appreciable information to what the clusters based on the vote percentage indicator variables convey.[20]

The second significant digits of the vote counts exhibit slightly more extensive clustering than do the vote counts' last digits, but much less than the vote percentage indicator variables. See Mebane (2015*b*) or the Election Forensics Toolkit for figures that show the clustering results for the second significant digits of the vote counts for the BNP and for the Awami League. For the BNP only one cluster of high second digit values overlaps with a cluster of high incremental fraud probabilities. For the Awami League none of the second digit clusters overlap with a cluster of high frauds probabilities. The two measures—the frauds probabilities and the second digits—do not appear to be much related in Bangladesh 2001, at least as far as the properties of their geographic distributions are concerned. If the frauds probabilities connect to genuine frauds, then the second digits are not detecting those frauds.

To emphasize the point that the second significant digits of the polling station vote counts probably are responding strongly to strategic behavior, we consider the conditional distributions produced when those digits are regressed on margins between the top candidates in each district. This kind of diagnostic, developed extensively in Mebane (2013*a*, n.d.), uses nonparametric regressions.[21] The margins of interest are the differences between the proportions of votes for the first-place and third-place party in each district and the differences between the proportions of votes for the second-place and third-place party. These margins relate to the proportion of votes for the first- and second-place

[20]The figures that show these clustering results may be viewed via the Election Forensics Toolkit.

[21]Nonparametric regressions are computed using the `sm` package (Bowman and Azzalini 2014, 1997) of **R** (R Development Core Team 2005).

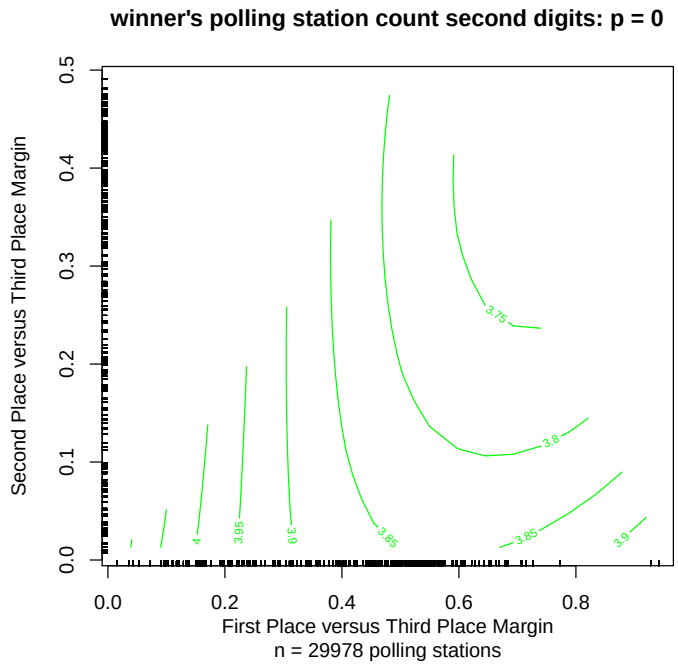
parties, respectively, that are being strategically switched by voters who are motivated by wasted vote logic (Cox 1994, 1997). Roughly speaking the rationale is that the larger each margin is then the more strategic vote switching occurs toward the two parties that are leading in each district. Mebane (2013*a*, n.d.) argues that these margins relate to the second significant digits of low-level vote counts in characteristic patterns.

Figure 11 shows patterns that match those typically seen when a certain kind of strategic voting occurs. In Figure 11(a) the outcome is the second digit in each polling station vote count for the first-place (winning) candidate in each district. In Figure 11(b) the outcome is the second digit in each polling station vote count for each district's second-place candidate. Rug plots along the edges of each graph show the locations of the margins for each district.

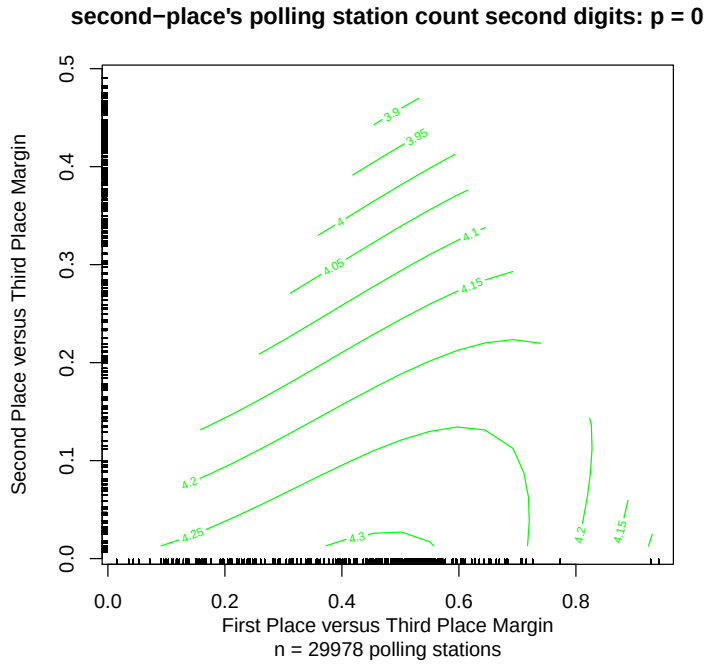
The regressions shown in Figure 11 are two-dimensional nonparametric regressions: both the first-versus-third and the second-versus-third margins are regressors. The mostly vertical contours of the conditional mean of the winner's second digits shown in Figure 11(a) imply that the digit means depend mainly on the first-versus-third margin, and the mostly horizontal contours of the conditional mean of the second-place finisher's second digits shown in Figure 11(b) imply that those digit means depend mainly on the second-versus-third margin. The conditional mean of the winning party's digits is always less than 4.187 and tends to decrease as the first-versus-third margin increases, as happens in simulations of strategic voting with multiple competitive parties, and then the mean increases slightly, as it does in data from Canada and Mexico (Mebane 2013*a*, n.d.). The conditional mean of the second-place party's digits is greater than 4.187 for a second-versus-third margin near zero, then decreases steadily as the margin increases. That is precisely the pattern seen in simulations with multiple competitive parties when there is no strategic voting (Mebane 2013*a*, Figure 2).

Figure 11: Second-digits Regressed on Inter-Party District Margins, Bangladesh 2001

(a)



(b)



Note: Nonparametric regression curve based on polling center data. "Margin" variables show the differences between the proportions of valid votes in each district for the first-, second- and third-place parties in each district. Rug plots show the locations of respective district margins. p-values (in each graph's title) report results of a test against a model in which there are no regression effects.

The pattern in which asymmetric strategic voting occurs—when the winning party appears to attract strategically switched votes but the losing party does not—is one that also occurs in data from Canadian elections (Mebane 2013*a*, n.d.). Nothing in the analysis in Figure 11 resembles what happens when a fraud based on coercion spanning the whole electorate is simulated (Mebane 2013*a*, n.d.). "Coercion" could mean any of voter intimidation, vote buying, ballot box stuffing or simply faked votes.

The Bangladesh 2001 election is complicated. Both frauds and strategic behavior likely occurred. Frauds occur in some areas: probably genuine fraudulent activities occur in the areas that have significantly high clusters of frauds probabilities; and genuine frauds are especially likely in areas where the frauds probability clusters overlap with the clusters that have high indicators for the occurrence of zeros and fives in the parties' rounded percentages. Our methods currently do not allow us to estimate the magnitude of the frauds in terms of the number of votes affected. The second significant digits of the polling station vote counts seem not to be much related to the frauds, but they very strongly suggest that a certain kind of strategic voting occurs. The strategic voting pattern is asymmetic: the winning candidate in a district attracts strategically switched votes but the second-place candidate does not. This strategic voting pattern is not compatible with some well-known theories of how wasted vote logic operates, but it is a pattern frequently observed in other countries in which election frauds are routinely believed to be rare and small.

3 Status of Election Forensics Methods

As the examples of South Africa 2014 and Bangladesh 2001 show, when adequate low-level aggregate data about an election are available, current election forensics methods allow one to reach extremely precise conclusions about whether and where election frauds occur. Here we have estimated the probability of frauds at individual polling stations. In other

cases, where suitable data existed, frauds probabilities have been estimated for individual ballot boxes (Mebane and Wall 2015). Such conclusions are always subject to the caveat that apparent frauds may really be consequences of strategic behavior, but that ambiguity can sometimes be mitigated by exploiting a multiplicity of statistics, as we have attempted to do here (and in Mebane 2015*b*). An election fraud will not necessarily trigger all of the statistics and tests, but we think a genuine fraud will in general set off many of them. Such reinforcement among diverse statistics and tests has informed the judgments we reach not only here about the elections in South Africa 2014 and Bangladesh 2001 but about elections in the other six countries on which analysis was conducted as part of the work for the grant that has supported this work (Mebane 2015*b*).

Election forensics as a field is at an early stage of its development, but already it is capable of supporting meaningful analysis. The set of methods we have used is not an exhaustive collection of all the methods that have been proposed. New methods continue to be developed and existing methods continue to be better understood. In this Guide we have illustrated one method for using geographic information to identify frauds, but elsewhere other methods have been considered (Mebane and Kalinin 2014) and many others are certainly conceivable. The core dilemma of trying to distinguish fraudulent from strategic behavior should be a central focus of future research.

There are also other important areas for further work in election forensics. Perhaps the most important outstanding problem, which we have largely ignored, is to adapt election forensics methods to features of the election system. Existing methods focus on the party or candidate that receives the most votes, sometimes labeling that party the "winner." But in election systems that use proportional representation, which are prevalent around the world, many parties are "winners" in the sense that many parties gain seats in the legislature as a result of votes cast in the election. Elections in several of our eight countries use proportional representation, but we have not done anything special to exploit or address that. Technology does not exist to do more than look at several of the

competing parties, as we have done. Some systems have district magnitudes greater than one with voting rules such as the single-non-transferable-vote, which causes complexities for election forensics. One of our countries, Libya 2014, is a case with such rules. Some of the methods we have described in this Guide do not work in such a system (see Mebane (2015*b*) for details). Other electoral systems exist (Taagepera and Shugart 1989; Shugart and Carey 1992; Cox 1997; Powell 2000), presenting a range of settings in which to tune election forensics methods. Different electoral systems present opportunities for a variety of strategic behaviors. The central dilemma that challenges election forensics—distinguishing election fraud from strategic behavior—will persist as we endeavor to develop methods to handle the full diversity of electoral systems.

4 Appendix: Digit and Distribution Approaches

2BL: second-digit mean. "Second-digit" refers to the second significant digit in each count to which the test is applied (for example, if the count is '1234,' then '2' is the second significant digit). Pericchi and Torres (2011) argue that if vote counts are produced in an unproblematic election, then the distribution of the second digits should be approximately be that implied by Benford's Law. The frequency with each digit occurs as the second significant digit according to Benford's Law is shown in Table 4. In fact the digits in vote counts do not have the distribution implied by Benford's Law, even though the second digits often do, a situation Mebane (2006, 2008) describes by referring to "second-digit Benford's Law-like" (or 2BL) distributions. (Pericchi and Torres 2011) argue that a kind of chi-square statistic applied to the distribution of the second digits of vote counts can detect election frauds, but many have expressed doubt about the efficacy of second digits as fraud detectors (Carter Center 2005; Shikano and Mack 2009; López 2009; Cantu and Saiegh 2011; Deckert, Myagkov and Ordeshook 2011; Mebane 2011). If the second digits occur with the frequencies given in Table

4, then their mean is $4.187 = \sum_{j=0}^{9} j r_j \equiv \bar{j}$, where r_j is the frequency for digit j. While frauds may cause the digits in vote counts not to have a 2BL distribution, Mebane (2013*a*, 2014, n.d.) shows that the second digits also respond strongly and with regular patterns both to the way voters are grouped into aggregation units (for example by gerrymandering) and to strategic behavior by voters (such as strategic voting). Conditional means (denoted $\hat{j}_x$) produced by nonparametrically regressing the second digits on 'x' variables such as the margins between candidates in each election district are needed to use the second digits to help diagnose whether frauds or strategies or both are occurring in an election. Value expected in the absence of frauds or strategic behavior: 2BL = 4.187.

Table 4: Frequency of Second Digits according to Benford's Law

digit	0	1	2	3	4	5	6	7	8	9
probability	.120	.114	.109	.104	.100	.097	.093	.090	.088	.085

LastC: last-digit mean. "Last-digit" refers to the last digit in each count to which the test is applied (for example, if the count is '1234,' then '4' is the last digit). Beber and Scacco (2012) argue that if vote counts are produced in an unproblematic election, then the distribution of the last digits should be uniform, which is to say that each each of the ten digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) should occur with frequency $\frac{1}{10}$. If the last digits occur with such frequencies, then their mean is $4.5 = .1 \sum_{j=0}^{9} j$. Beber and Scacco (2012) exclude counts less than 100 when assessing the last digit distributions. Using data from Russian elections Mebane (2013*b*) shows that the last digits can appear to have a uniform distribution even when substantial election frauds can be demonstrated by other means—in such cases the digits are likely faked. But such fakery requires near total control over the reported results. Value expected in the absence of frauds or strategic behavior: LastC = 4.5.

C05s: mean of a binary variable indicating whether the last digit of vote count for the referent party or candidate is zero or five. As a special case of the Beber and Scacco (2012) argument, the expected value for the mean of this indicator variable in the absence of frauds is C05s = 0.2.

P05s: mean of a binary variable indicating whether the last digit of the rounded percentage of votes for the referent party or candidate is zero or five. If agents need to signal that they have committed fraudulent acts then these digits may be especially likely to appear, as has been amply demonstrated to occur in Russian elections (Mebane and Kalinin 2009*a*,*b*; Kalinin and Mebane 2011; Shpilkin 2011; Mebane 2013*b*; Rundlett and Svolik 2015). A mean for the variable that indicates whether the rounded percentages end in zero or five that is larger than 0.2 suggests such a signal-related manipulation. Value expected in the absence of frauds or strategic behavior: P05s = 0.2.

DipT: test of unimodality. Motivation for the test of unimodality comes from the argument that multimodality (multiple distinct peaks) in the distribution of turnout proportions indicates irregularities (Myagkov, Ordeshook and Shaikin 2009; Levin et al. 2009). A theoretical argument about the dynamic process in which collective choices occur also asserts that under a wide range of conditions distributions not only of turnout but of winners' vote shares should be unimodal (Borghesi and Bouchaud 2010). The test is from Hartigan and Hartigan (1985). The statistic is the p-value for a null hypothesis that the distribution is unimodal. Very small p-values give reason to reject the hypothesis. Value expected in the absence of frauds or strategic behavior: DipT $>$ .05.

Skew: skewness. If arguments against multimodality are sharpened into an argument that the distribution of turnout proportions should be Normal, then skewness should be zero. A Normal distribution is symmetric. Skewness is a measure of asymmetry. A symmetric distribution, and in particular the Normal distribution, has skewness zero.

If skewness is significantly different from zero, there may be problems with the election. Klimek et al. (2012) point out that skewness alone is not necesarily an indicator for election frauds, but significant nonzero skewness along with other indicators (such as kurtosis) can suggest that frauds occurred (positive or negative skewness). Value expected in the absence of frauds or strategic behavior: Skew = 0.

Kurt: kurtosis. If arguments against multimodality are sharpened into an argument that the distribution of turnout proportions should be Normal, then kurtosis should be three. Kurtosis measures the rate at which the frequency of values that differ from the mean declines as values more and more distant from the mean are considered. A Normal distribution has kurtosis equal to three. If kurtosis is significantly different from three, there may be problems with the election. Klimek et al. (2012) point out that kurtosis alone is not necesarily an indicator for election frauds, but kurtosis significantly different from three along with other indicators (such as skewness) can suggest that frauds occurred (excess kurtosis). Value expected in the absence of frauds or strategic behavior: Kurt = 3.

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