Modeling Survival Data Using Frailty Models.

David D. HANAGAL. Boca Raton, FL: CRC Press, 2011, xix + 314 pp., $89.95 (H), ISBN: 978-1-4398-3667-5.

It is common that individuals identical in measurable attributes such as age, gender, race, and treatment may differ in many unmeasured ways in their life experience, including onset of disease, drop out from school, default, end of marriage, and ultimately death. Frailty offers a convenient way to model unobservable attributes or factors that would modify an individual's tendency toward these events. Since the first introduction of frailty in survival analysis (Vaupel, Manton, and Stallard 1979), frailty survival models, which include frailty as an unobserved random proportionality factor to modify the hazard function of an individual or a group of related individuals, have found applications in areas such as demography, medical science, finance, engineering, environmental science, astronomy, and justice. However, there are only a handful of books available that cover frailty survival models in a systematic way (Hougaard 2000; Duchateau and Janssen 2007; Wienke 2010). David Hanagal's book is a much needed attempt to cover new topics that are not available in the existing books and to bridge the recent research activities with applications.

The book is logically divided into three parts: basic introduction (Chapters 1–3), shared frailty models (Chapters 4–10), and bivariate frailty models (Chapters 11–14). It is worth mentioning that the R code for the data analyses is also included.

The introduction focuses on survival models for independent data and also describes eight key data examples referenced and analyzed throughout the text. The author opts for providing interesting but realistic (in terms of structure) datasets, which would facilitate or encourage readers to try out the newly learned methods. By the conclusion of Chapter 1, readers should have a grasp of some basic but fundamental concepts in survival analysis, including survival functions, hazard functions, and censoring. Chapter 2 studies some commonly used parametric survival models, including the exponential, Weibull, extreme value, and gamma models, as well as the likelihood-based approach for statistical inference. Chapter 3 moves on by introducing the common nonparametric and semiparametric techniques for modeling independent survival data. The Kaplan–Meier estimation of a survival function, log-rank and Wilcoxon tests, and the Cox proportional hazards model are succinctly described here.

Chapter 4 begins Part II, which is devoted to shared frailty models, wherein related subjects share a common frailty term. In this chapter, the author formally introduces the frailty concept and sheds light on the implications of frailty. The conditional parameterization with various frailty distributions and the marginal parameterization in the sense of copulas (though the author does not explicitly dwell on it) are noted. Chapter 5 extends Chapter 4 by explicitly giving various representative frailty distributions. The "classical" frailties, such as gamma, positive stable, and power variance frailties, as well as the newly emerged compound Poisson frailty are presented in an easily accessible fashion. In Chapter 6, the likelihood-based approach is described for making inference based on shared frailty models. A general expectation–maximization (EM) algorithm and a modified EM algorithm tailored for gamma frailties are clearly presented. Chapter 7 goes on by applying the method described in Chapter 6 to analyze almost all of the datasets introduced in Part I of the book. The presented R code and the output highlight the usefulness of the chapter (and maybe the book!).

The author slightly changes gears in Chapter 8 by testing the existence of frailty or heterogeneity based on shared frailty models, focusing on the likelihood ratio and score tests. Chapter 9 moves on by "stacking" the frailties described in Chapter 5 on top of existing bivariate exponential and Weibull models and derives a variety of new bivariate survival functions. This chapter has certainly benefited much from the expertise of the author, who has a long list of publications in the area. Chapter 10 presents a time-dependent version of frailty, namely, a frailty constructed from the Levy process. The time-dependent notion, as argued by the author, is highly biologically driven. For example, some disease risks are acquired at birth possibly due to genetic makeup, whereas some risks may accumulate throughout life depending on lifestyle and environmental exposure. A Levy-process-based frailty (and several other versions) as presented in this chapter can be beneficial to the practitioners working in the related fields.

The final four chapters, constituting Part III of the book, shift focus to bivariate frailties and other correlated frailties, which distinguish this book from the other existing frailty books. Chapter 11 introduces bivariate correlated frailties for a pair of correlated subjects (e.g., monozygotic or dizygotic twins). The author briefly describes the well-known confounding issue between the frailty and covariate effects, and proposes a general copula-based model to circumvent the confounding issue. Several approaches for statistical inference, including a two-stage approach (commonly used in the copula setting), a basegroup estimation method, a likelihood-based EM algorithm, and profile estimation, are discussed.

Chapter 12 continues on by presenting an interesting result that allows one to construct a bivariate extension of some univariate family distributions. By this construct, the correlation coefficient of the bivariate frailty distribution can be explicitly modeled as a parameter, which might be appealing to practitioners, in particular those who take on genetic association studies. Chapter 13 moves beyond the bivariate paradigm and studies multivariate frailty models. Additive frailty models, representing various degrees of unobserved exposure (e.g., pedigree in genetic studies), highlight the power of using frailty models to study genetic linkage of diseases. Chapter 14 returns to study the identifiability of bivariate frailty models, which in my view, seems to be out of place. It should either be delegated to the Appendix or be placed right after Chapter 11 where the bivariate frailty model is first introduced.

In terms of strengths, the author is to be applauded for giving a comprehensive and easily accessible summary of the development of frailty models and the applications in various fields. The author should also be complimented for updating readers with the (relatively) new correlated frailty models, derived from the traditional univariate and shared frailty models. The author seems to have a strong preference for genetic applications. An appealing feature of the book is the inclusion of datasets and the R code, which will be welcomed by readers with different backgrounds.

My criticisms are minor. Given the vast nature and the rapid development of the topic area, the author must have made some tough decisions about coverage. That said, I first note that the discussion of univariate survival models is rather narrow and only encompasses the Cox proportional hazards model. There is no mention of competing models, including the semiparametric accelerated failure time model, additive hazards model, proportional odds model, and semiparametric transformation model. Possibly, the author relies on readers to have acquired such knowledge from other sources/textbooks. Second, the methods for drawing statistical inference are primarily likelihood based. The widely used estimating equation approach is seldom referred to in the text. Large sample theories are also lacking, making it difficult to theoretically evaluate the performance of the described methods. Third, the text falls entirely to the Frequentist paradigm and completely ignores the advance of Bayesian approaches in the relevant areas, especially in the area of spatial statistics. Lastly, the book does not offer any problem sets, and therefore may not be suitable for being taught in a classroom setting.

In summary, the book reflects a strong appreciation of mathematical development and applications of frailty survival models. The author's many years of research experience in the topic area helps to disseminate the results to a general audience, for whom the book is intended. An intermediate statistics graduate student with some general background knowledge in univariate survival analysis should be able to come away with some solid understanding of the presented materials. The book can also be used as a handy reference book for researchers with a variety of applications in mind.

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Numerical Methods in Finance With C++.

Maciej J. CAPINSKI and Tomasz ZASTAWNIAK. New York: Cambridge University Press, 2012, x + 166 pp., $39.99 (P), ISBN: 978-0-521-17716-0.

This short book in the Mastering Mathematical Finance (MMF) series addresses quantitative finance using the popular computer programming language