Agenda:

1. p-values
2. classical linear model
3. method of moments

Recap:

Linear model w/ Gaussian/normal errors

$\underbrace{y}_{n\times 1} \mid \underbrace{X}_{n\times p} \sim N(X\beta^*, \sigma^2 I_n)$ for some vector of "true" regression coeff's $\beta^* \in \mathbb{R}^p$

Under the lin. model w/ Gaussian errors: $\hat{\beta} = (X^T X)^{-1} X^T y$

$\hat{\beta} - \beta^* \mid X \sim N(0, \sigma^2 (X^T X)^{-1})$

Hypothesis testing under normality:

want to test $H_0: \beta_j^* = 0$

reject $H_0$ if $\left| \frac{\hat{\beta}_j - \cancel{\beta_j^*}^{\,0}}{SE[\hat{\beta}_j]} \right| > t_\alpha$ (rejection statement for hypothesis test)

Type 1 error rate: $t$ is chosen so that

$P_0\left( \left| \frac{\hat{\beta}_j - \cancel{\beta_j^*}^{\,0}}{SE[\hat{\beta}_j]} \right| > t_\alpha \right) \le \boxed{\alpha}$ — usually 0.05

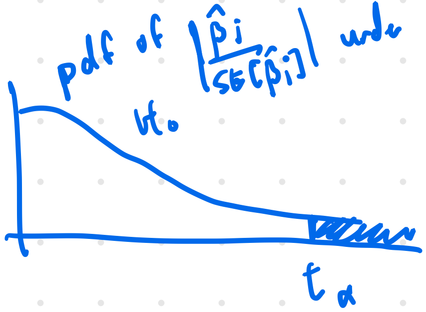
($t$ is the $1-\alpha$ quantile of $\left|\frac{\hat{\beta}_j}{SE[\hat{\beta}_j]}\right|$ under $H_0$)

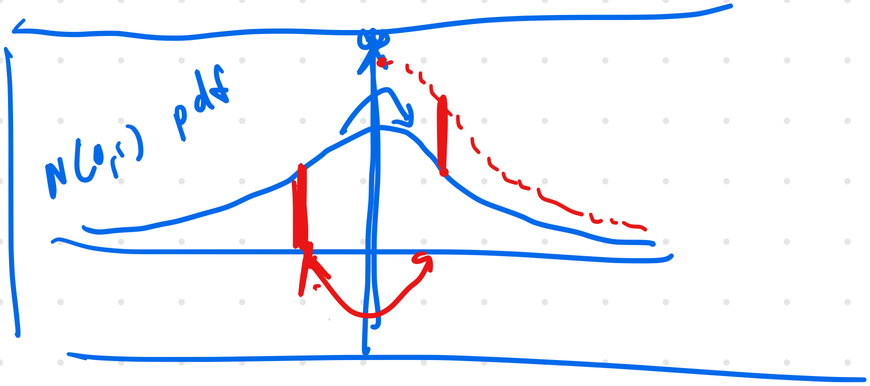
The correct value of $t_\alpha$ is about 2 (for $\alpha \simeq 0.05$).

The rejection statement is equivalent to

reject if $\underbrace{1 - F\left( \left| \frac{\hat{\beta}_j}{SE[\hat{\beta}_j]} \right| \right)}_{\text{p-value}} \le \alpha$, where $F$ is the CDF of a $\left| \frac{\hat{\beta}_j}{SE[\hat{\beta}_j]} \right|$ under $H_0$

In the z-test $F$ is the CDF of $|z|$, where $z \sim N(0,1)$

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Classical linear model:

Condition (linearity): $\underset{n\times 1}{y} = \underset{n\times p}{X}\underset{p\times 1}{\beta^*} + \underset{n\times 1}{\epsilon}$

" definition of error terms as $\epsilon = y - X\beta_*$ "

exogeneity : $E[\epsilon \mid X] = 0$

spherical error variance : $E[\epsilon\epsilon^T \mid X] = \sigma^2 I_n$

} comparable to Gaussian err terms condition

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Aside: covariance of random vector

If $x \in \mathbb{R}$,

then $var[x] \overset{\circ}{=} E[(x-\mu)^2]$

$\mu \overset{\circ}{=} E[x]$

If $x \in \mathbb{R}^n$ is random vector

then $Var[x] = E[(x-\mu)(x-\mu)^T]$

$\mu = E[x]$

$[Var[x]] = E[(x_i - \mu_i)(x_j - \mu_j)]$

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$E[\epsilon\epsilon^T \mid X] = \sigma^2 I_n$ { homoskedasticity: $E[\epsilon_i^2 \mid X] = \sigma^2$ for all $i = 1, \dots, n$
no serial correlation: $E[\epsilon_i \epsilon_j \mid X] = 0$ for $i \neq j$

<u>method of moments:</u>

$\underset{p\times 1}{E[x_i} \underset{1\times 1}{\epsilon_i]} = E[x_i(y_i - x_i^T \beta^*)]$

$= E[E[x_i \epsilon_i \mid X]]$ (Tower property (law of total expectations))

$= E[x_i \cancel{E[\epsilon \mid X]}]$

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